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A MATHEMATICAL FRAMEWORK FOR AN IMPROVED SEARCH MODEL

Jeffrey Nicoll

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1801 N. Beauregard Street, Alexandria, Virginia 22311-1772

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PREFACE

This paper was prepared for Mr. Thomas Hafer, Deputy Director Advanced Systems Technology Office, ARPA, under an ARPA Project Assignment on Analysis and Model Development. Additional technical cognizance and direction have occurred through Dr. John Brand, U.S. Army Research Laboratory, S^3I Special Projects Office.

I would like to acknowledge the following individuals for their suggestions and comments made during the development of this work and during the review process: Mr. John D'Agostino, Dr. John Brand, Mr. David Dixon, Dr. Grant Gerhart, Dr. Walter Lawson, Mrs. Luanne Obert, Mr. Gary Witus, and Mr. Andy Wong.

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SUMMARY

This paper provides an extension of the model for human search performance currently used in Department of Defense applications. The proposed model is termed "neoclassical" because it is intended to be only a small departure from the classical model. It incorporates a broad mathematical framework that addresses complex modeling issues without becoming so cumbersome that it cannot be used in large-scale modeling simulations such as Janus or CASTFOREM. The framework was developed as part of the Army Target Acquisition Model Improvement Program (TAMIP). As a part of TAMIP, measures of target attractiveness (how strongly a target attracts the eye) were developed to extend the contrast- or temperature-based models for the target signature. The neoclassical approach connects these to the search and target acquisition modeling domains.

The classical field-of-view search model is represented by a single-exponential, two-parameter model. The predicted probability of detection is:

$$P_D(t) = P_\infty \left(1 - e^{-t/\tau} \right) . \qquad \text{(S-1)}$$

P_∞ is the fraction of an ensemble of observers that will eventually detect the target. For each observer who will see the target the time constant, τ, describes the detection rate. P_∞ and τ are typically estimated by straightforward heuristic extensions of the static performance model. The effects of competing targets and clutter are included in an ad hoc fashion based on field experience.

The classical model is simple and simplicity is an important attribute of any model used as a component of overall systems modeling. However, the details of human search performance experiments are highly complex and cannot always be satisfactorily represented by Eq. (S-1). In addition, the two parameters of the classical model, while they provide adequate guidelines to performance, do not allow for detailed and quantitative description of clutter backgrounds; in particular, they may not address the problems associated with inconspicuous targets or severe clutter. It is therefore desirable to place the classical model into a more general mathematical framework. This will provide a better understanding of the limitations of the classical approach as well as giving a more fundamental method for estimating the crucial parameters of the model. The neoclassical

model provides a general framework that permits the addressing of a broader domain of search applications without violating the need for simplicity.

When we apply existing mathematical models, for the most part, they rapidly become too complicated to be validated in detail or applied in practice. In this work the emphasis is on reducing the complexity of the formulation. A reasonable starting point is the Markov process. A Markov process consists of a number of states with specified transitions between the states. In the context of search, one could assign a separate state to each fixation point in the field of view. The transitions would then correspond to the probabilities that the observer moves his gaze from one point to another during search.

The probability of detection predicted by a Markov process model is a linear combination of exponential terms (one for each state) rather than the single exponential in the classical model. This approach can be useful if the number of states used is small. Using a separate state for each point in the scene is not practical because the number of states is too large. On the other hand, the classical model is a two-state Markov model: the first state represents a continuing search, while the second state represents detecting the target.[1] This does not provide enough flexibility to describe the effects of multiple targets and clutter. The goal of this paper is to strike a balance between these extremes, retaining sufficiently many states to accurately describe the search without unnecessarily complicating the results.

The neoclassical model assigns a separate state to each target or target-like clutter point; these are described in terms of the target attractiveness or target signature metrics. Assigning a state to every target candidate is necessary if the effects of competing targets and clutter are to be handled. In general, this would lead to as many exponents as target-candidates and would be too unwieldy for most applications. However, by representing the large number of state-to-state transition rates by a few average transition rates, we can reduce the number of exponents to a small number.

For field-of-view search, this paper develops and justifies a model for which three exponents are sufficient, independent of the number of targets and clutter points:

$$P_D(t) = \sum_{i=1}^{3} e_i \left(1 - e^{-\lambda_i t}\right) . \qquad \text{(S-2)}$$

1 There are, in fact, two exponents in this Markov description of the classical model; the second exponent is zero and describes the fact that the sum of the probability of detecting and not-detecting the target is unity.

The value of the exponents and the amplitude coefficients depend on the relative attractiveness of the target of interest and the competing targets and clutter. For searches with long overall search times (> 10 seconds), one eigenvalue is much smaller than the others and dominates P_D for long times; the three-exponent model reduces to the single-exponent, classical model with a time constant determined by the competing targets and clutter. The remaining terms act as small corrections for short times. Thus, the important difference between the neoclassical and classical models is not the difference between one exponent and three exponents; there is nothing magical about three exponents. Rather, the neoclassical model provides the modeler with a systematic method for computing search times from target and clutter metrics describing the scene.

If the approach is extended from single region or field-of-view to a multiple region or field-of-regard search, there would be an increase in the number of states and a corresponding increase in the complexity. However, a hierarchical analysis can reduce the number of required exponents. Each field of view within the field of regard is treated as a "super-target." The search proceeds among the super-targets and then at a finer level of detail, within each super-target. The field-of-view results are used to compute the best single-exponent description to use within the field of view. This gives a two-exponent approximation for field-of-regard search. The same approach can be used to describe regional search: for example, a scene divided into distinctive regions such as tree lines, roads, and meadows. A similar approach provides a three-exponent approximation for wide versus narrow field-of-view search. For these applications, the eigenvalues are closer and no single exponent approximation suffices.

The neoclassical model is simple enough to be falsifiable; that is, it has precise enough predictions that a comparison with experimental data will reveal its applicability and shortcomings. For each application the neoclassical framework provides a straightforward approach. Descriptions of target and clutter signatures are used to predict the time constants and the probability of detection. The model replaces the single exponential classical search model with two or three exponentials depending on the application. This minor increase in complexity can be straightforwardly included in large-scale simulations such as Janus and CASTFOREM. The neoclassical model contains many of the insights gained from the classical model and from recent work in human visual system modeling. It is robust without being overly complicated and may prove to be a useful data analysis tool as well as providing support to large-scale simulations.

I. INTRODUCTION

A. BACKGROUND

The search model developed in this paper is an extension of the model currently used for modeling human search performance for Department of Defense applications. An excellent review of the standard model is given in Howe (1993). The new model is termed "neoclassical" because it is only a small departure from the classical model described by Howe. It deals with issues that are not well addressed by the classical formulation without becoming so mathematically cumbersome that it cannot be used in large-scale modeling simulations such as Janus or CASTFOREM. It provides a general mathematical framework in which search modeling questions can be posed and studied.

The currently used classical model, while generally successful, uses an ad hoc procedure for handling clutter and multiple targets. A more general mathematical framework provides a systematic treatment of these issues. This framework makes contact with recently developed psychophysical models of the cueing[1] and detection processes and to address the results of human search and detection experiments.

The model has been developed as part of the Army Target Acquisition Model Improvement Program (TAMIP) and supplements program work done on human vision modeling. As a part of this program, new metrics have been devised for clutter, target signatures, and target attractiveness[2] [O'Kane et al. (1993), D'Agostino et al. (1993), Kowalczyk et al. (1993), Witus (1993), Doll (1993)]. The target attractiveness metrics use detailed computational vision models of the human visual system, as well as target and background metrics to provide a measure of a particular target's strength in the search

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1 "Cueing" is defined for the purposes of this paper as that which brings the observer's attention to a target or clutter object for the purpose of considering it as a target. It therefore goes beyond a simple fixation on the target; it is apparently close to the concept of "finding" as introduced by D'Agostino et al. (1993) but does not require a decision to be made. One may be cued to the target by a particular feature of the target or simply by its brightness.

2 Target attractiveness is a measure of the propensity of a target to attract the eye of the observer and may, in principle, be distinguished from target signatures that are used to measure detection or recognition performance once the target is considered by the observer. The distinction will only be valuable when target attractiveness metrics have been separately validated. In the interim, any target signature metric can be considered as a candidate for measuring target attractiveness.

process. The neoclassical connects the target attractiveness models to the search and target acquisition modeling domains.

The neoclassical model has the capability of describing:

- Multiple targets. For each target considered separately, the other targets represent distractions and slow the search and detection processes. Targets are considered to be essentially independent; target interactions can be partly dealt with by introducing target arrays as "super-targets."
- False alarms. Clutter objects and targets are described in a single uniform formalism permitting parallel treatments of false and correct detections.
- Clutter. The clutter in a scene can act both as a distraction and as a source of false detections. The uniform treatment of clutter objects and targets allows for both effects.
- Different models of detection. The search and detection models are decoupled. This permits different models of the detection process within the same search paradigm. The model can describe detection processes that permit multiple looks at the target as well as models that restrict detection to a few visits.
- Different "initial conditions" of search. These can affect the predicted probability of detection, particularly for short times.
- Different models for P_∞. P_∞ is the fraction of an ensemble of observers who will eventually detect the target. Several different methods are discussed including quitting, use of the observer ensemble, restricting visits, nonexponential and nonstationary methods.
- Single region, field-of-view search, multiple region, field-of-regard search, and wide-field versus narrow field search can all be incorporated into the neoclassical framework.

The neoclassical model introduces a general framework for the discussion of search and detection processes. As such it may appear to be mathematically complex; however, in many cases of interest the model reduces to the limiting case of a single exponent and provides simple answers to search problems.

B. CLASSICAL FIELD-OF-VIEW SEARCH MODEL

The classical field-of-view search model is represented by a single-exponential, two-parameter model. The search process is assumed to be a series of random glimpses. The eye moves around the field of view in jumps called saccades. Between saccades the eye, in particular the foveal region of highest acuity, is fixated on a particular region of the scene. The combination of saccade and fixation defines a glimpse. Under the usual

approximations, the predicted probability of detection P_D as a function of time, t, is given by an expression of the form

$$P_D(t) = P_\infty (1 - e^{-t/\tau}) \ . \tag{I-1}$$

In (I-1), the single amplitude coefficient, P_∞, is usually interpreted as the fraction of the observers who will eventually detect the target [but see Rotman (1989)]. For each such observer who will see the target, the time constant, τ, describes the rate of the detection process— $1/\tau = P_{glimpse}/T_{glimpse}$, where $P_{glimpse}$ is the probability that the target is found during a single glimpse and $T_{glimpse}$ is the mean time of a glimpse (taken as approximately 0.3 seconds).

Eq. (I-1) can be used to describe experimental data in which case P_∞ and τ represent two adjustable fitting parameters. For modeling purposes, they are predicted. The number of resolved cycles on target, N, is computed from the static model; P_∞ is then given by:

$$P_\infty = \frac{\left(\frac{N}{N_{50}}\right)^E}{1+\left(\frac{N}{N_{50}}\right)^E}; \quad E = 2.7 + 0.7\frac{N}{N_{50}} \tag{I-2}$$

where N_{50} is the number of resolved cycles on target for 50 percent probability of detection. The exponent E was determined by experimental data fits. N_{50} is adjusted to account for task difficulty as in Table I-1 (Howe, 1993).

Table I-1. Approximate Value of N_{50} in Different Levels of Clutter

Task	Example	N_{50}
Highly Conspicuous Target	Bright source, movement, zero clutter	<< 0.5
Low Clutter	Target in field, on road	0.5
Medium Clutter	Tank in a desert of tank-sized bushes	1.0
High Clutter	Vehicle in array of similar vehicles	2.0

The value of the single exponent, τ, for low and medium clutter is taken as:

$$\tau \approx 3.4/P_\infty \qquad P_\infty < 0.9 \tag{I-3a}$$

$$\tau \approx 6.8\, N_{50}/N \qquad P_\infty \geq 0.9 \tag{I-3b}$$

The classical model is simple and simplicity is an important attribute of any model used as a component of overall systems modeling. However, the details of the time dependence of human search performance experiments are highly complex and cannot always be satisfactorily represented by such a model. For short times, Eq. (I-1) implies

$$P_D = P_\infty \frac{t}{\tau} \ . \tag{I-4}$$

That is, the probability of detection increases linearly with time. Not only is the increase linear but the amplitude is required to be P_∞/τ; these parameters more naturally are associated with the long time behavior of the detection probability. It is, therefore, not surprising that in real human performance data, any attempt to model with two parameters both the short and long time behavior may be inadequate. A single exponential may be a good fit for long times, but the behavior near $t = 0$ does not agree with the extrapolation to short times of the best medium to long time exponential fit [Blecha et al. (1991), Nicoll (1992), Cartier et al. (1994)]. Initial slopes that differ from that predicted by Eq. (I-4) and even zero initial slopes are often observed.

Similarly, the treatment of targets and clutter through the adjustment of N_{50}, while adequate for many modeling applications, does not allow for detailed and quantitative description of multiple targets in cluttered backgrounds; a description that begins with the target and background characteristics (as measured by appropriate metrics) and then calculates the effect on detection times would be more desirable.

C. MATHEMATICAL APPROACH: MARKOV MODELS

The principal task of formulating a more general framework for search is not due to an absence of possible mathematical models to apply. However, for the most part, these models rapidly become too complicated to be validated in detail or applied in practice.

A reasonable starting point is the Markov process [evidence that search is a Markov process is discussed in Harris (1993)]. A Markov process consists of a number (K) of *states*; there are *transitions* between the states that occur at specified rates. Mathematically, a Markov process is described by a set of K linear differential equations describing the probability of each state. The goal of this paper is to identify reasonable choices for the states and transition rates, but all the mathematics has the same form and is briefly reviewed here to set the context.

Denote the probability of being in a state (named F_i) as f_i, and the transition rate from the jth state to the ith state as T_{ij}. Then Markov process equations are

$$\dot{f}_i = \sum_j T_{ij} f_j - (\sum_j T_{ji}) f_i \tag{I-5}$$

where the dot denotes time differentiation. These linear equations are straightforward to solve in the abstract by diagonalizing the K × K transition rate matrix, and determining the K eigenvalues and eigenvectors. The general solution describes the probabilities of being in a particular state as a linear combination of K exponentials depending on K initial conditions. The more detailed the description of the process (the larger the number of states), the more transition rates and initial conditions are required.

The classical model can be cast as a Markov process with two states: continuing search and detection. Denote the probability of having detected the target as p, and the probability of still searching as s. Then the Markov process equations are:

$$\begin{aligned} \dot{s} &= -s/\tau \\ \dot{p} &= +s/\tau \end{aligned} \tag{I-6}$$

where τ^{-1} describes the transition rate from search to detection. The solution is:

$$\begin{aligned} p(t) &= 1 - s(0)e^{-t/\tau} \\ s(t) &= s(0)e^{-t/\tau} \end{aligned} \tag{I-7}$$

where s(0) is the probability that the observer is searching at t = 0. The probability that he has already detected, p(0), is p(0) = 1 – s(0). The usual case is p(0) = 0 and s(0) = 1.

There are several points to note. First, the general Markov theory provides two exponents; but in this case, one of the exponents is zero: $s(t) + p(t) = 1 = e^{0t}$. Second, in this Markov representation, $P_\infty = 1$. The model does not include a prediction of P_∞. Several different methods of determining P_∞ are discussed later in the paper. Third and most importantly, the only adjustable parameter is τ. There is no natural way of introducing the effects of clutter or competition from other targets except by directly modeling those effects in τ itself. Thus, this model is too restrictive and is not rich enough to describe targets and clutter in a natural way.

At the other extreme is a Markov process that assigns a different state to every possible fixation point in the image (in practice, the search field could be divided into regions of foveal size, ≈ 1 deg^2). From a purely mathematical point of view, this "foveal fixation state" model is as simple as the classical model and it contains all the flexibility required to describe multiple targets and clutter, but it is impractical in application. The number of separate sites to be represented may be extremely large. For unaided eye search

the number of states would be over 10^4 and the number of transition rates would be $\approx 10^8$. There would be K exponents and K initial conditions to be determined; this is *a priori* implausible. Even for aided search with a restricted field of view, say 35 deg^2, the number of independent terms is daunting. While of interest for particular aspects of carefully controlled experiments, it would not be suitable for "hand work" or force-on-force simulations. For the analysis of human performance experiments, there are too many adjustable coefficients for such a model to be useful as a data reduction tool. Thus, the two-state classical model is too restrictive and the foveal fixation state model is too complex—a balance must be sought which preserves the simplicity of the classical model while permitting a more fundamental description of the competition between targets and clutter.

D. REQUIRED ELEMENTS OF THE NEOCLASSICAL SEARCH MODEL

The neoclassical model describes three distinct search processes:

- Attending to the target of interest. "Attending" implies: (1) the observer is looking at the target; and (2) the observer is considering the target and deciding if it is a target.
- Attending to other objects. These objects may include other targets or clutter objects. If the observer's attention is being given to these objects, they act as distracting elements to the task of detecting a specified target.
- Wandering. That is, moving from fixation point to fixation point without considering the fixation points as serious candidates for targets.

It might seem at first glance that only two processes are needed: attending to the target and not attending to the target. However, if this approach is taken, it is difficult to consider multiple targets and to maintain the separation between being distracted by other possible targets and simply examining the scene as a whole. Three processes seem to be the minimum number necessary to capture the essential features of search. The interaction of these processes is indicated schematically in Fig. I-1. Attending to the target of interest is denoted P_0; attending to the set of other target candidates is denoted by $\{P_i\}$; wandering through the image is denoted W.

The search process controls the detection process since detection occurs during the process of attending to the target; detection is separated from the search process and assumed to have no direct effect on search. This approximation allows multiple targets and a variety of detection models to be incorporated.

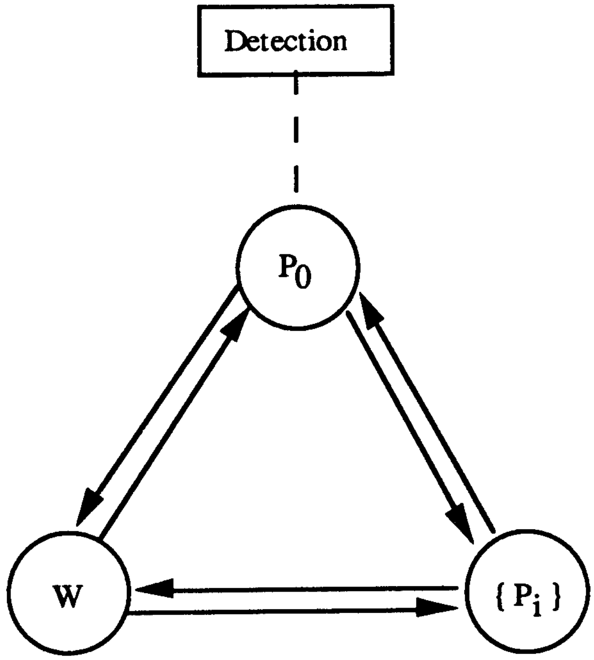

Figure I-1. Illustration of the Three Search Processes and Detection

Choosing the processes does not completely determine the model. For a scene with N possible targets (or points of interest), it is natural to define a Markov model with N + 1 states: one state for general wandering and one state for attending to each of the points of interest. In general, this leads to a solution with N + 1 exponents. This is more practical than the foveal fixation state model but still may be clumsy for multiple targets in cluttered backgrounds. On the other hand, a model with fewer states cannot easily represent multiple target cases—each target should be considered as a separate state.

This conflict between simplicity and the need to represent each target candidate is resolved in the neoclassical model by making simplifying assumptions about the transition matrix. These assumptions lead to a probability of detection function with three exponents corresponding to the three general processes discussed above.

By extending from a single exponent to three exponents, the complexity of the model is increased slightly, but there are clear benefits. The neoclassical model provides a uniform treatment of targets and clutter; each target or clutter object is separately described. Instead of an ad hoc procedure for calculating the appropriate N_{50} value for a given level of clutter, the equations themselves determine the time constants of the search process.

For searches with long overall search times (> 10 seconds), one eigenvalue is much smaller than the others (longer time constant) and dominates P_D for long times; the

three-exponent model reduces to a single-exponent, classical model with a time constant determined by the competing targets and clutter. The remaining terms typically act as small corrections for short times (< 5 seconds). This may resolve the discrepancies between the classical model and experimental data for short times, and can be used to predict the survivability of targets with limited exposure times, but for most applications, the single-exponent form is an excellent approximation.

Thus, the important difference between the neoclassical and classical models is not the difference between one exponent and three exponents; there is nothing magical about three exponents. Rather, the neoclassical model provides the modeler with a systematic method for computing search times from target and clutter metrics describing the scene.

The following sections will develop the model in detail. Section II will provide the mathematical details to flesh out the description of the search and detection process illustrated in Fig. I-1. Section III will provide a series of examples that illustrate the implications of the model. The neoclassical model does not prescribe a particular method of obtaining $P_\infty < 1$. Section IV discusses several approaches: (1) quitting the search process prior to detection; (2) an observer ensemble as in the classical model; (3) models that restrict the number of visits; and (4) nonexponential detection processes.

Section V extends the discussion from single region or field-of-view search to multiple region, field-of-regard search. This can be treated in two different ways. The first method defines intermediate steps between the simple three-exponent model and the "foveal fixation state model," adding states and increasing the number of exponents. The second approach uses a simplified version of the three-exponent field-of-view model to provide a two-exponent field-of-regard representation and a related three-exponent model for wide versus narrow field-of-view search. Section VI is a summary of the work.

Appendix A provides some auxiliary discussion of technical details of the assumptions and approximations of the neoclassical model. Appendix B gives some further limiting cases of the neoclassical model. Appendix C discusses the higher order correlation functions of the detection processes as induced by the search; this provides a measure of the statistical independence of the probabilities of detection for single targets. Appendix D discusses nonstationary extensions of the model within the neoclassical framework.

II. THE NEOCLASSICAL FIELD-OF-VIEW SEARCH MODEL

This chapter will describe the mathematical details of the field-of-view search model; subsequent sections will give applications of the model and extend it to other search problems. The exposition is fairly formal to make all of the mathematical approximations clear.

A. STRUCTURE OF THE MODEL

Assumptions of the neoclassical model are:

(1) The model uses a Markov representation of search among N "points of interest" that include both targets and target-like clutter points. The non-target-like fixation points are all combined into a single state called "wandering." This parameterization of the problem provides a unified treatment of targets and clutter. However, the use of a Markov representation implies that the search process has no memory.[1] Sites that have been visited once may be visited again. This simplification will be considered further as the model is developed.

(2) The detection process for each point of interest is independent of any other and of the search itself.[2] This separation of the detection and search processes is artificial but flexible. Different detection mechanisms can be included within the search model; in fact, a different detection mechanism can be considered for each object. The separation of detection from search implies that the act of detection does not directly change the search process. One can imagine that a master search program directs the process and occasionally issues a detection report and continues searching. Although the search process has no memory itself, the detection process may; for example, the detection process may restrict detection to a fixed number of visits. On subsequent visits, no detections are made.

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1 One may note as an aside that one can partly escape from the Markov no-memory restriction by considering a pseudo-Markov process that removes targets or clutter objects from the list of points of interest as decisions are made about them; the search model is then "rebooted" with the revised list and specified initial conditions. This extension is not difficult but will not be addressed in this paper.

2 By assuming that different targets will be handled independently, target array effects and target-target interactions in the detection process are not completely modeled. Some aspects of target dependence may be included by defining points of interest corresponding to target arrays as "super-targets." In this case the detection process can be represented as finding all of the array as a part of a single process.

(3) The search model itself does not demand a particular interpretation or implementation of the long time detection probability, P_∞. Since the detection mechanism is mathematically decoupled from the search process itself, a variety of methods can be employed. Several methods of introducing a $P_\infty < 1$ are discussed.

(4) In its simplest form, the model requires three parameters for each target or target-like object to be represented. The first two parameters represent the average attractiveness of the particular target compared to all other target candidates; these are parameters of the search process itself and correspond to the probability being cued to the target from the wandering state and from another target candidate, respectively. The third parameter governs the detection process and corresponds to the probability of detection given a cue. These parameters should be calculable from a validated human vision model.

The next section implements these assumptions mathematically.

B. MARKOV SEARCH MODEL

The search process consists of sequences of fixation points, divided into two classes:

- "Random" wandering fixations. These fixations are assumed never to lead to target detections. From a wandering fixation, the observer can move to another similar wandering fixation or to a point of interest.
- Points of interest (POI) that may be either targets or clutter objects. The observer will be assumed to dwell on these points for several saccade times. During this time, the detection process for that object is engaged and a decision to declare the object a target or non-target can occur. The observer may leave a particular point of interest by returning to the wandering state or by "jumping" to another point of interest. Subsequent returns to the same POI are allowed.

Figure II-1 shows a representation of the search pattern envisioned. Long, wandering saccades are followed by a series of short saccades during which the observer attends to a possible target. To make this picture mathematically precise, we must define the various states of the Markov process. Assigning a different state to each fixation point would be unworkable for data analysis or simulations—approximations are needed.

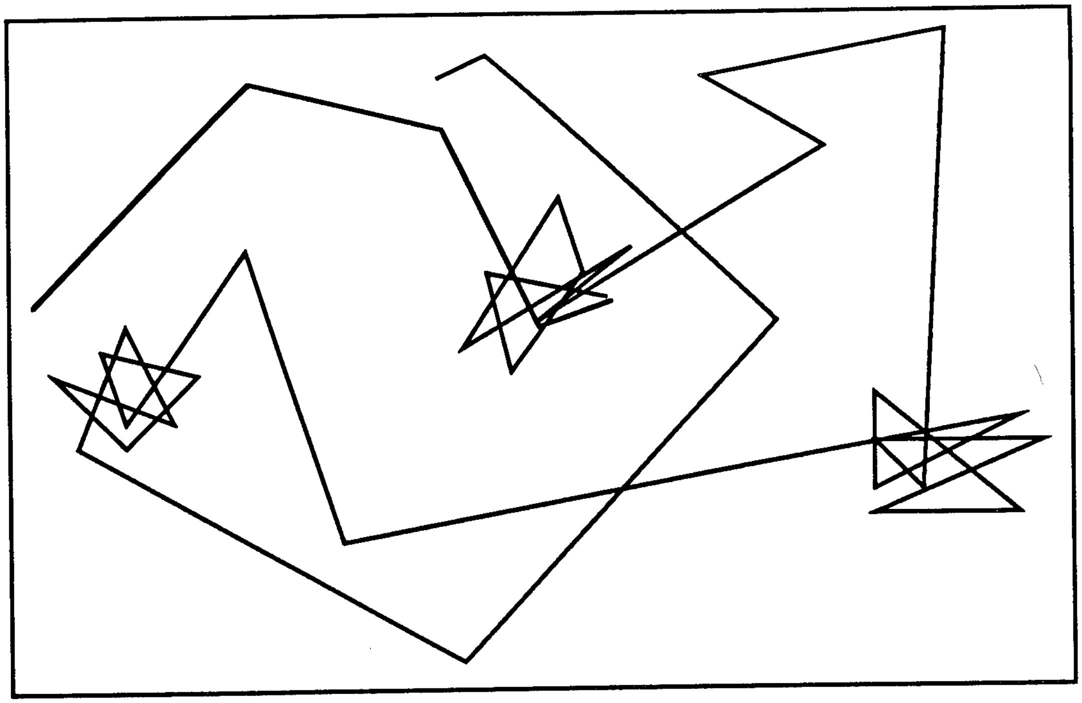

Figure II-1. Search Pattern for a Single Region

The first approximation is to collapse all of the different wandering fixations into a single wandering state;[3] each of the N points of interest is then assigned a separate state (see Fig. II-2; the transitions between states are not shown). This amounts to assigning Markov states based on the presumed cognitive state of the observer: he is either examining one of the N target candidates or just wandering.

The probability that the observer is in the wandering state is denoted by w; p_i denotes the probability that the observer is fixated on the ith point of interest, P_i. It is convenient to define p as the sum of all the p_i.

$$p = \Sigma\, p_i = \text{probability that the observer is fixated on some POI.} \qquad \text{(II-1)}$$

Since the observer has to be somewhere, $w + p = 1$.

Transitions between any two states are permitted: wandering state to a POI, POI to the wandering state, and POI to POI. The transition rate from the wandering state to P_i depends on the average "attractiveness" of the point of interest and is denoted S_i. Since all

3 By collapsing the wandering fixations into a single state, all the transition probabilities into and out of the wandering states become averages over the fixation location. In other words, transition rates that depend on the eccentricity (angular distance) of the target to the fixation point are averaged over eccentricity. Precisely how this average is performed depends on the search strategy—that is, how the random fixations are distributed. They may be uniform over the scene or concentrated (treeline search).

the different wandering fixation points have been collapsed into a single state, S_i is an average over the different eccentricities between P_i and the possible wandering state fixation points. The transition rate from P_i to the wandering state is W_i and again represents an average rate. The transition rate from P_j to P_i is denoted J_{ij}.

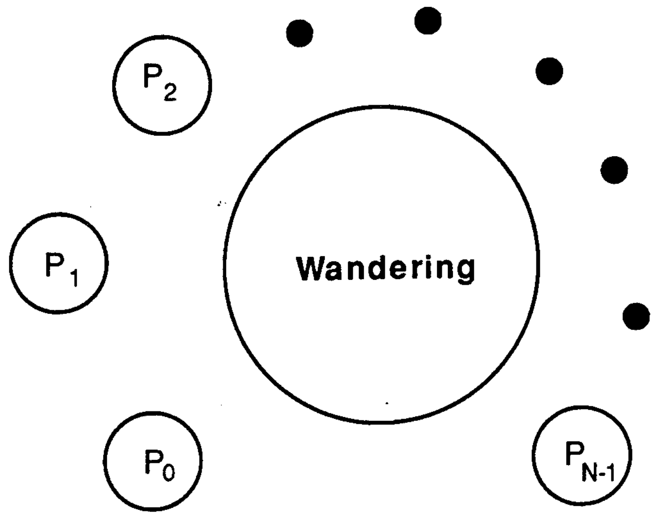

Figure II-2. Markov State Diagram

Assuming that the average time between saccades while in the wandering state is T_{sac}, then S_i is proportional to the probability of cueing to P_i.

$$S_i \approx P_{cue(i)} / T_{sac} \quad . \qquad \text{(II-2a)}$$

$P_{cue(i)}$ is the probability that the observer is "cued" to the point of interest: fixated on it and attending to it as a target; it is provisionally assumed to be calculable from a human vision performance model.[4] The sum of all the S_i rates determines the overall rate of leaving the wandering state and attending to some point of interest. Define the "total attractiveness of the points of interest," S, by:

$$S = \Sigma S_i \quad . \qquad \text{(II-2b)}$$

Similarly, the jump transitions from POI to POI are determined by the probability of cueing to the ith state given that the observer is attending to the jth state, $P_{cue(i)|cue(j)}$. Using a typical dwell time on a POI, T_{dwell}, J_{ij} would be:

$$J_{ij} \approx P_{cue(i)|cue(j)} / T_{dwell} \quad . \qquad \text{(II-2c)}$$

4 In the absence of a human visual performance model, these parameters can be modeled as free parameters or assumed to be related to static model performance, for example, proportional to P_∞.

With this notation, the Markov process equations become:

$$\begin{aligned} \dot{w} &= -S\,w + \sum_i W_i p_i \\ \dot{p}_i &= S_i\,w - W_i p_i + \sum_j J_{ij}\,p_j - \left(\sum_j J_{ji}\right) p_i \;. \end{aligned} \tag{II-3}$$

These equations are still too complex, requiring the diagonalization of an (N+1) × (N+1) matrix. Therefore, additional simplifications are required.

The second approximation parallels the first. Using a single average transition rate from the wandering state to each POI is equivalent to assuming that only the destination of a saccade matters, not the origin (see Appendix A for a discussion of these approximations). Applying this to the jump transition rates, we set $J_{ij} = J_i$, so that the jump transition rate depends only on the point of interest jumped to. Eqs. (II-3a) reduce to:

$$\begin{aligned} \dot{w} &= -S\,w + \sum_i W_i p_i \\ \dot{p}_i &= S_i\,w - W_i p_i + J_i\,p - J\,p_i \end{aligned} \tag{II-4a}$$

where

$$J = \sum_i J_i \quad = \text{ total jumping rate} \;. \tag{II-4b}$$

Although the use of an average jumping transition rate reduces the number of parameters, there are still N + 1 exponents. In the third step of the approximation, we assume that a single average rate describes the transitions from each point of interest to the wandering state: $W_i = W$ (this is partly relaxed below).

$$\begin{aligned} \dot{w} &= -S\,w + W\,p \\ \dot{p}_i &= S_i\,w - W p_i + J_i\,p - J\,p_i \;. \end{aligned} \tag{II-5}$$

The equations are now markedly simpler. As is shown below, these lead to at most three exponents to describe the search and detection processes. The only parameters needed are

W = average rate of leaving a POI to wander

S_i = average rate of entering the P_i from wandering

J_i = average rate of entering the P_i from another POI.

These rates are all experimentally measurable in human performance experiments and have a direct physical meaning. One expects that S_i and J_i are related. For example, the simplest assumption is that J_i is proportional to S_i:

$$J_i = \kappa S_i \quad ; \quad J = \kappa S \tag{II-6}$$

where, presumably, $\kappa \leq 1$, reflecting the fact that it is less likely that the observer will leave one point of interest to jump to another than it is for the observer to move from the wandering state to that point of interest.[5]

The approximation of equal values of W_i simplifies the mathematics but is also supported by experiment [Rotman et al. (1993)]. If required by the further study of the experimental data, different values of W for different classes of POI can be accommodated by increasing the number of states and exponents.[6]

The mean time to dwell on or examine a particular point of interest is:[7]

$$T_{exam,i} = \frac{1}{W+J-J_i} \quad . \tag{II-7}$$

A more attractive POI will have a larger J_i and will have a longer mean examination time.

In the neoclassical model, targets (single or multiple) and clutter objects are treated symmetrically; this unified treatment of the points of interest permits a unified treatment of probability of detection and false alarms. The cueing probability for clutter objects can, in principle, be calculated by the same human vision model used for targets. If in a particular scene there are discrete target-like clutter objects that are predicted or found to be sources of false alarms, the cueing probability of those objects can and should be explicitly calculated. In this case, one can use the detection process modeling to compute the false alarm performance in a manner completely analogous to the probability of detection of targets. In some cases, a detailed description of the scene is unavailable or inappropriate to the task and an overall average effect of the clutter is needed. One approach is to depend on

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5 In some circumstances this might not be the case. The average over possible fixation points used to define the S_i may provide a different distribution of eccentricities with respect to the target than the more restricted average over the locations of the other points of interest used to define the J_i. For particular target arrays, this may make the rate of jumping to the target from a point of interest large compared to cueing to it from the wandering state. See also Appendix A.

6 One of the field-of-regard, or multiregion, search models discussed in Section V uses a distinct wandering state for each region.

7 This can be understood by considering the search equation for an observer that is known to be examining the particular POI, P_i. Thus, while in that state $w = 0$, $p = p_i$. The equation for the decay of the probability out of the state is then given by:

$$\dot{p}_i = +J_i p_i - J p_i - W p_i \quad .$$

In any single visit the probability that the observer stays on the site decays with a simple exponential with a time constant given by Eq. (II-7).

the computational vision models [Doll (1993), Witus (1993)]. These models use a signal-to-noise formulation to compute the cueing probability; there is also a probability of a false cue. If the false targets are not analyzed directly as targets, the probability of cueing to a clutter point could be analyzed by taking for each clutter point the false cue probability associated with the threshold used to define the cueing probabilities for targets, $P_{false\ cue}$.

$$P(\text{Clutter})_{cue} = P_{false\ cue} \quad . \tag{II-8}$$

This false cueing probability applies to each foveal region in the image; the larger the image the greater the probability that the observer is falsely cued to some part of the image. Thus, the contribution to S ("total attractiveness of the points of interest") is expected to be proportional to FOV $P_{false\ cue}/T_{sac}$ where FOV is the size of the field of view in foveal fixation regions. A similar contribution is made to J proportional to κ FOV $P_{false\ cue}/T_{sac}$. This provides a lumped parameter representation of the effect of the clutter that does not require a point-by-point analysis. The exact proportionality constant must be determined by validation of the human performance models coupled with the search model.

In the last resort, ad hoc contributions to S could be tabulated for different scene types and circumstances. This would be analogous to Table I-1 for different values of N_{50} and should be avoided if a fundamental quantitative description of clutter is desired.

C. MATHEMATICAL DEVELOPMENT: SEARCH

A Markov process describes a never-ending sequence of transitions among the states. There is no unique path of state transitions; if one considers the average over all possible paths taken by an ensemble of observers, the fraction of the observers in a particular state approaches an equilibrium value, described by the Markov process equations. For p(t) and w(t) the solutions of Eqs. (II-5) are:

$$p(t) = \frac{S}{R} + \left(p(0) - \frac{S}{R}\right) e^{-R\,t} \tag{II-9a}$$

$$w(t) = \frac{W}{R} + \left(w(0) - \frac{W}{R}\right) e^{-R\,t} \tag{II-9b}$$

where $R = S + W$ and w(0) and p(0) are the initial probabilities that the observer is wandering or is attending to some point of interest, respectively [note that w(t) + p(t) = 1, for all times]. Eq. (II-9) describes a population of observers each following his own path through the states; for long times, a fraction S/R of these observers will be attending to one of the POI while the remaining fraction W/R will be wandering. The exponent R governs the relaxation between the initial and equilibrium values.

The probabilities for the POI obey:

$$p_i(t) = p_i^{eq} + (p_i(0) - p_i^{eq})e^{-(J+W)t}$$

$$- \frac{[p_i^{eq}(J+W) - w(0)S_i - p(0)J_i]}{(J - S)}[e^{-Rt} - e^{-(J+W)t}] \qquad \text{(II-9c)}$$

where the long time fraction of the observers fixated at P_i (the equilibrium value of p_i) is:

$$p_i^{eq} = \frac{S_i W + J_i S}{R(W+J)} \qquad \text{(II-9d)}$$

and $p_i(0)$ is the probability that the ith point of interest is being attended to at $t = 0$. The additional exponent $W + J$ appears in this expression and governs the relaxation to equilibrium among different points of interest.

Equations (II-9) have *three* eigenvalues: R, $W + J$, and 0. The zero eigenvalue expresses the fact that the sum of the entire probability, $\varepsilon(t) = w(t) + p(t)$ is a constant, $\varepsilon(t) = 1$. When detection is considered, these eigenvalues will shift and the 0 eigenvalue will become the most important exponent for the description of detection.

D. DETECTION REPRESENTATION

Detection is assumed to be a process that happens during a fixation at a point of interest but that does not change the search process directly. Having visited a target candidate POI, the search returns to the wandering state or other points of interest, possibly returning later to the target candidate. Detection at P_i is essentially independent of detection at P_j and can even be modeled by different detection mechanisms.[8] The basic detection mechanism discussed here is an exponential detection process. This is simple and fits naturally into the Markov representation of the search; it can be used to generate other models of the detection process. To begin with, P_∞ will be assumed to be unity: all the targets are eventually found. Different methods of introducing $P_\infty < 1$ are discussed in Section IV.

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8 To be technically precise, the search process is completely independent of the detection processes but the detections are not independent of the search. The probability of detection of a particular target is independent of the detection or nondetection of all of the other targets. However, the higher order detection correlations (such as the probability that neither target A nor target B has been detected) are linked by the search process, simply by the fact that the observer cannot be two places at once. For example, if a particular search path spends all of its time on one target, it cannot spend any time on another. This produces a correlation between target detections when the average over all search paths is made. These correlations will be discussed in Appendix C.

Consider a specific target (the 0th). The detection process is modeled as an exponential process obeying a stochastic differential equation:

$$\dot{P}_D(t) = \alpha_0(1 - P_D(t))\,\eta_0(t) \tag{II-10}$$

where α_0 is the detection rate for the 0th target and $\eta_0(t)$ is a random function of time determined by the search process:

- $\eta_0 = 1$ if the search state is P_0, i.e., the observer is attending the 0th target. In this case the probability of the observer declaring the object to be a target increases as determined by the exponential detection equation.
- $\eta_0 = 0$ otherwise, i.e., the observer's attention is elsewhere; the probability of declaring the 0th target remains at the value reached at the end of the last visit to the target.

The rate α_0 governs the rate of increase of detection probability while attending to the target. It is proportional to the probability of detecting the target given that the observer is cued to the 0th target: $\alpha_0 \approx P_{det|cue(0)}/T_{sac}$.

For a particular search path, the solution of Eq. (II-10) is

$$P_D(t) \quad = \quad 1 - e^{-\alpha_0 T(t)} \tag{II-11a}$$

where T(t) is the time spent attending to the target in question:[9]

$$T = \int_0^t \eta\,(s)\,ds \quad . \tag{II-11b}$$

For any particular path through the states of the system, the probability of detection is a single exponent model in terms of the time on target, T. Since the particular search path taken by the observer is not known, the expected value of the probability of detection is computed by averaging over all possible search paths:

$$P_D(t) \quad = \; \langle 1 - e^{-\alpha_0 T(t)} \rangle \tag{II-12}$$

where the < ... > averages over the Markov search process. The three exponents arise when the average over the possible search paths is taken.

---

9 As will be shown below, the neoclassical model straightforwardly extends to any detection model that depends on the time on target. For nonstationary models, one may include detection processes that depend upon the time on target for each individual visit to the target. The total time on target, T, is the sum of the visit times, T_j, where T_j is the time on target for the jth visit.

It is shown in Appendix A that this average can be computed by modifying the Markov process by adding a fictitious absorbing state to P_0. That is, a term is added to the p_0 equation representing a transition to a "detection state." The equations become (defining $\varepsilon = w + p$)

$$\dot{\varepsilon} = -\alpha_0 p_0 \tag{II-13a}$$

$$\dot{w} = W\ p - S\ w + (W_0 - W)p_0 \tag{II-13b}$$

$$\dot{p}_0 = S_0 w + J_0 p - J p_0 - W_0 p_0 - \alpha_0 p_0 \tag{II-13c}$$

with $P_D(t) = 1 - \varepsilon(t)$.

In Eq. (II-13), the symmetry between the particular state of interest, P_0, and the other points of interest is broken by the introduction of the α_0 term. Since the state has already been singled out, the value of W for that state can be assigned a state-specific value, W_0, without a large cost in increased mathematical complexity. Allowing for $W \neq W_0$ introduces some extra algebraic complexity; however, by retaining it, Eq. (II-13) has the most general form possible for a three-exponent model.

The additional state is termed fictitious because it was introduced simply to perform the average in Eq. (II-12) and does not represent a genuine state of the search process. As discussed in Appendix C, multiple target correlations are computed by introducing additional fictitious states. Of course, it is possible for an absorbing state to be real. If, for example, the observer quits whenever he detects the target, then the absorbing state is real. The distinction between fictitious and real absorbing states can be subtle; a discussion is given in Appendix A.

Since the detection process is assumed to be independent for each point of interest, each point of interest satisfies an equation of the same form. The probability of detection of the ith point of interest is:

$$P_{D,i}(t) = P_D(t, \alpha_i, S_i, J_i, W_i) \ . \tag{II-14}$$

In many cases, these probabilities are approximately independent (but see Appendix C) and the usual rules for combining independent probabilities apply.

The detection and search processes combined are illustrated in Fig. II-3 (transitions between states are not shown, the individual detection processes are denoted Det_i). Attached to each point of interest is a detection process. Within the exponential detection

model, Eq. (II-10), each detection process is coupled to the search process through a detection parameter, α_i, indicated by the dashed lines.

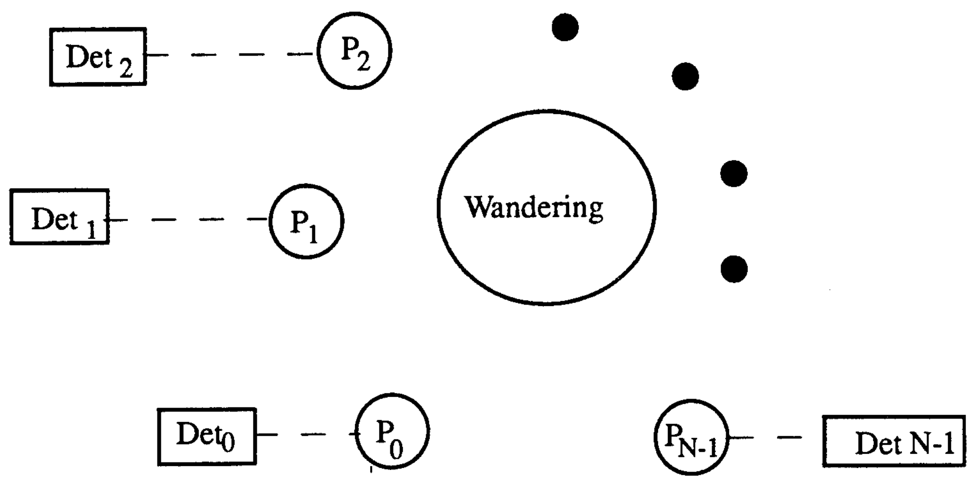

Figure II-3. Combined Detection and Search Processes

The relationship of the detection processes at each point of interest and the total search process is analogous to an old-fashioned time-sharing system. The search process governs the visits of the observers to each point of interest; it represents the scheduling algorithm of the time-sharing system. While at the ith POI, the ith target's own detection process program runs for an average time of $1/(W + J - J_i)$. The values of S_i are analogous to the operating system's priority given to the individual task. The J_i represent "message passing" in the time-sharing system in that one computation initiates another.

For *each* point of interest considered separately, the other points of interest collapse into a single distracter state and act as a load on the observer, slowing down the performance. From the point of view of each process, the system of Fig. II-3 reduces to the simpler diagram given in Fig. II-4; the many states of the problem have been effectively reduced to three. The approximations made in the neoclassical model are essential if this simplification is to be obtained.

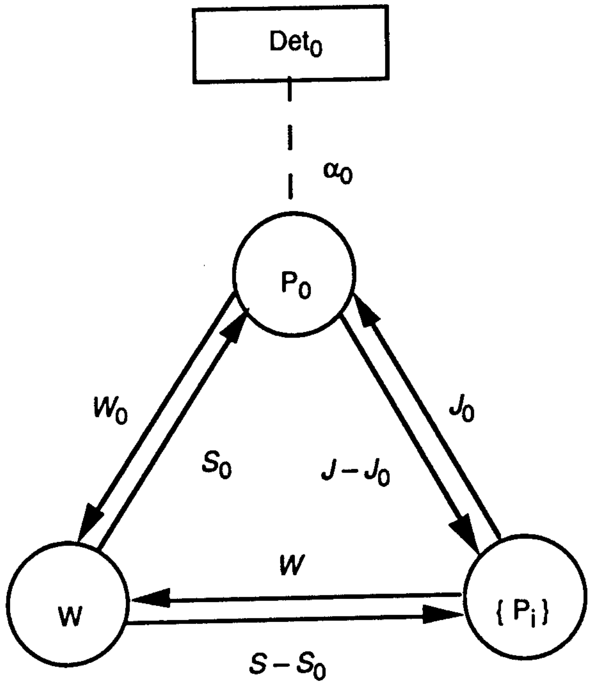

Figure II-4. Effective Processes as Seen by Each Detection Process

E. MATHEMATICAL DEVELOPMENT: DETECTION

The three eigenvalues are now obtained from the solution of the cubic equation derived from the matrix of coefficients of Eq. (II-13),

$$\lambda\,[(\lambda - R)(\lambda - \hat{W}_0) - \delta W \Delta_0] - \alpha_0\,[S_0 W + J_0 S - J_0 \lambda] = 0\ , \qquad \text{(II-15a)}$$

where for notational convenience and to compress some algebraically lengthy expressions

$$\hat{W}_0 = W_0 + J + \alpha_0 ; \quad \delta W = W_0 - W\ ; \ \Delta_0 = S_0 - J_0\ . \qquad \text{(II-15b)}$$

The discussion of the neoclassical model does not depend on an explicit solution of the cubic equations. However, for illustrative purposes it is convenient to have specific forms in mind. The simplest approach is to consider a perturbation expansion around some limiting case. For inconspicuous targets, one may assume that the probability of cueing to a particular target is small; that is, S_0 and J_0 are small. For small cueing rates, the following suffices (compare to the unperturbed eigenvalues of R, $W_0 + J$ and 0):

$$\lambda_1 = R + \frac{(S_0 - J_0)\,(\alpha_0 W + R\,\delta W)}{R\,(R - \hat{W}_0)} \qquad \text{(II-16a)}$$

$$\lambda_2 = \hat{W}_0 - \frac{\alpha_0 [S_0 W + J_0 S - J_0 \hat{W}_0]}{\hat{W}_0 (R - \hat{W}_0)} \tag{II-16b}$$

$$\lambda_3 = \frac{\alpha_0 [S_0 W_0 + J_0 S_0]}{R (W_0 + \alpha_0 + J_0)} \quad . \tag{II-16c}$$

The solution for $\varepsilon(t)$ is a linear combination of the three exponents that depends on the initial conditions. The computations are an exercise in linear algebra:

$$P_D(t) = \sum_{i=1}^{3} e_i [1 - e^{-\lambda_i t}] \tag{II-17a}$$

$$e_i = \frac{\alpha_0 [\frac{S_0 W + J_0 S}{\lambda_i} - S_0 w(0) - J_0 p(0) + p_0(0) (\lambda_i - R)]}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} \tag{II-17b}$$

where in Eq. (II-17b) i, j, k = 1, 2, 3 and permutations. The initial probabilities of being in the wandering state, attending to some POI and attending to the 0th target, are w(0), p(0), and $p_0(0)$, respectively. The model also predicts other quantities that can be directly measured from experimental data. The probability of detection on a single visit is[10]

$$P_{Visit} = \frac{\alpha_0}{W_0 + J - J_0 + \alpha_0} \quad . \tag{II-18a}$$

As α_0 gets large, the probability of detection on the first visit approaches unity. Each visit in a Markov process is independent of any other; thus, the mean number of visits is:

$$N_{Visit} = \frac{1}{P_{Visit}} = \frac{W_0 + J - J_0 + \alpha_0}{\alpha_0} \quad . \tag{II-18b}$$

The mean time to detect is the most significant single parameter in the model.

$$\langle t_D \rangle = \sum_{i=1}^{3} \frac{e_i}{\lambda_i} \quad . \tag{II-19}$$

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10 This is computed from the probability of not detecting during a single visit of length T, given by $e^{-\alpha T}$, and the probability density of visits of length T, $e^{-(W_0-J-J_0)T}(W_0-J-J_0)dT$, and then integrating over all visit lengths.

It can be written in terms of the mean time until the first visit,[11] T_1, the mean time between subsequent visits, T_s, and the mean number of visits:

$$T_1 = \frac{R\,(1-p_0(0)) - w(0)(S_0 - J_0)}{S_0 W + J_0 S} \tag{II-20a}$$

$$T_s = \frac{R\,(W_0 + J - J_0) - W_0(S_0 - J_0)}{(W_0 + J - J_0)\,(S_0 W + J_0 S)} \tag{II-20b}$$

$$\langle t_D \rangle = T_1 + (N_{Visit} - 1)\, T_s + \frac{1}{\alpha_0} \tag{II-20c}$$

$$\langle t_D \rangle = T_1 + \frac{1}{\alpha_0 p_0^{eq}} \quad . \tag{II-20d}$$

The particularly simple forms of the time to detect given in Eq. (II-20c) and Eq. (II-20d) are general results for any Markov model when expression is in terms of the average number of visits, time between visits, and the equilibrium probability. Note that the dependence of the mean time to detect on the initial conditions is entirely contained in T_1 and the time subsequent to the first arrival depends on the detection rate α_0 and the equilibrium probability of examining the target. The latter depends on the competing targets and clutter as shown in Eq. (II-9d). The mean time to detect (t_D) can also be written in terms of the eigenvalues:

$$\langle t_D \rangle = \frac{1}{\lambda_3} + \frac{1}{\lambda_2} + \frac{1}{\lambda_1} - \frac{S_0 w(0) + J_0 p(0) + R\, p_0(0)}{S_0 W + J_0 S} \quad . \tag{II-21}$$

If one of the eigenvalues is much smaller than the others, it dominates the mean time to detect.

Finally, the mean value of the length of a visit (T_{Visit}) to the target is:

---

11 As α_0 becomes large, the probability of detection on the first visit is unity and the mean time to detect is the time until first visit. Since the search process is Markov, T_s is the mean time between the first and second visits, or between the second and third visits, etc. It can be computed from the formula for time to first visit by setting the initial conditions appropriately; namely, $p_0(0) = 0$ and w(0) equals the mean value after visiting the target:

$$w(0) = \frac{W_0}{W_0 + J - J_0} \quad .$$

$$T_{\text{Visit}} = \frac{1}{W_0 + J - J_0} \quad . \tag{II-22}$$

These expressions for the number of visits, first time to the target, time between visits, length of visit, and mean time to detect can be directly measured in human search experiments and used to establish the parameters of the neoclassical model and to provide validation of its predictions.

F. NEOCLASSICAL MODEL FIELD-OF-VIEW SEARCH RESULTS

Although Eqs. (II-17) appear cumbersome in the abstract, they are not complicated in the final application. The consequences of the neoclassical model are developed by examples in the next section, but the following provides the main results:

(1) The different exponents can be assigned approximately distinct roles. Any such assignment is necessarily rough since the probability of detection is the sum of all three terms.

λ_1 describes transient behavior in the search process itself, balancing the time spent on targets with the time spent wandering. It does not depend strongly on the detection rate, α_0. The time associated with λ_1 is on the order of the dwell time on a target during search.

λ_2 describes the time spent dwelling on the target for the purposes of detection. It depends strongly on the detection rate, α_0, increasing with increasing α_0. The time associated with λ_2 may therefore become shorter than a dwell time if the target is detected (when cued) in less than a typical dwell time.

λ_3 describes the time of the search and detection process as a whole. It depends on the cueing rates, S_0 and J_0 as well as detection rate, α_0. If a target is hard to cue to or hard to detect, then λ_3 is small, corresponding to a search and detection process that is long compared to the typical dwell time.

(2) Searches can be divided into long and short search depending on the relative sizes of the exponents.

For a long search $\lambda_3 << \lambda_1, \lambda_3 << \lambda_2$. The search will take many target dwell times, either because of the intrinsic difficulty of the target or due to competing clutter. A long search might be one of greater than 10–20 seconds duration.

For a short search $\lambda_3 \sim \lambda_1, \sim \lambda_2$. The search will take only a few target dwell times, either because of the intrinsic properties of the target or due to a lack of competing clutter. A short search might be one of 5–10 seconds duration.

(3) For long searches:

The neoclassical model is well represented by a single exponent, λ_3. The neoclassical model goes beyond the classical model by providing a basis for calculating the value of the exponent that is based in the target signatures. All the clutter, competing targets, and detection rate information are represented by λ_3. The remaining exponents act as correction terms.

(4) For short searches there is little distinction between short times and times in the asymptotic region. A single exponent may be a reasonable numerical approximation but the correction terms now are relatively important at all times.

III. EXPLORATION OF THE NEOCLASSICAL MODEL

This section will illustrate the neoclassical model's general expressions given in Section II with a number of limiting cases and numerical examples.

A. LONG vs. SHORT SEARCHES

This section illustrates the distinction between long and short searches. For long searches, it is shown that a single exponent approximation is excellent. As the mean search time decreases, the deviations from the single exponent approximation become larger. Four cases are explored numerically; for simplicity, the examples use $W_0 = W$ and $J = J_0 = 0$. In all cases, we assume that an observer spends about 1 second dwelling on a point of interest, $W = 1$; that the observer spends 2/3 of his time examining target candidates and 1/3 wandering, $S = 2$; and that the characteristic time for detecting the target if attending to it is 1/2 second, $\alpha_0 = 2$. The value of S_0 is varied:

Case A. Long Search. There are 10 equally strong points of interest so that for the target of interest, $S_0 = 2/10$.

Case B. Moderate Search. There are 5 equally strong points of interest so that for the target of interest, $S_0 = 2/5$.

Case C. Short Search. There are 2 equally strong points of interest so that for the target of interest, $S_0 = 1$.

Case D. Very Short Search. There is only one point of interest, $S_0 = 2$.

Table III-1 gives the exponents for the three cases. Note that λ_1 and λ_2 do not strongly depend on the value of S_0. In all three cases λ_3 is smaller than the other exponents and is much smaller for Cases A, B, and C. Thus, it dominates the mean time to detect [Eq. (II-21)] and a single exponent approximation is an excellent description of the probability of detection. When there is any substantial initial probability of attending to the target of interest, the single exponent form must include the effects of the initial visit. Using the $S_0 << W$ limits of the exact expressions:

$$P_D = 1 - (1-p_0(0)P_{visit})e^{-\lambda_3 t} \quad . \tag{III-1}$$

At $t = 0$, the single exponent approximation gives $P_D = p_0(0)P_{visit}$; this corresponds to the fact that the neglected fast exponents, λ_1 and λ_2, die out by the end of the first visit. The single exponent approximation represents this by jumping discontinuously to the level expected at the end of the visit.[1]

Table III-1. Example Exponents and Time to Detect

Description	Case A: S_0=0.2	Case B: S_0=0.4	Case C: S_0=1.0	Case D: S_0=2.0
λ_3 (sec^{-1})	0.046	0.095	0.27	1
λ_2(sec^{-1})	2.61	2.43	2.0	1
λ_1(sec^{-1})	3.35	3.48	3.73	4
t_D: Distracted (sec)	22.5	11.25	4.5	NA
t_D: Wandering (sec)	21.5	10.25	3.5	1.25
t_D: On Target (sec)	7.5	3.75	1.5	0.75
$1/\lambda_3$(sec)	21.82	10.55	3.73	1
δt: Distracted (sec)	0.68	0.70	0.77	NA
δt: Wandering (sec)	−0.32	−0.30	−0.23	0.25
δt: On Target (sec)	0.23	0.23	0.26	.42

In addition to the exponents, Table III-1 gives the slow time constant $1/\lambda_3$, which is related to mean time to detect in the single exponent approximation of Eq. (III-1) by:

$$\langle t_D \rangle \approx (1-p_0(0)P_{visit})/\lambda_3 \quad . \tag{III-2}$$

The table also gives the mean time to detect as calculated from Eq. (II-21) and the difference, δt, between the Eq. (II-21) and Eq. (III-2) for three initial conditions; (a) distracted: the observer begins the search attending to a different point of interest; (b) wandering: the observer begins in the wandering state; and (c) on target, the observer begins attending to the target of interest.

If the observer begins in the distracted state, the detection is delayed about 3/4 second for Cases A, B, and C (for Case D, the observer cannot be distracted since the target is the only point of interest). For long searches this is a very small fraction of the total search time but it becomes a larger fraction as the search time shortens.

If the observer begins in the wandering state, the search times are shortened by about 1/4 second for Cases A, B, and C; this begins as an insignificant fraction of the mean

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1 This clearly will cause the mean time to detect to be artificially shortened for $p_0(0) \neq 0$.

time to detect for Case A and increases to about 8 percent for Case C. In Case D, the approximations made to go from the full results to the single exponent approximation are breaking down, but the search time is apparently lengthened by 20 percent.

Finally, if the observer begins on target, the single exponent approximation (as used in Eq. (III-1)) underestimates the mean time to detect by about 1/4 sec for Cases A, B, and C and 1/2 sec for Case D.

These results are reinforced by examining the predicted probability of detection. Figure III-1a shows the results for Case A. Three curves are plotted for the single exponent approximation, a wandering start and a distracted start. The curves are essentially identical except for the shortest times. Figure III-1b shows the results for Case B; the curves are still nearly identical although the differences at shorter times are larger. Figure III-1c shows the results for Case C; the curves now are distinctly different. Finally, Fig. III-1d shows the single point of interest Case D. The large differences shown in Case D are partly a result of using Eq. (III-1) to define the single exponent approximation.[2]

These examples show that, for long searches, the details of the neoclassical model prediction only affect the short time performance.[3] As the mean search time is reduced, the short time effects become relatively more important and for very short searches they are as important as the long time behavior. While an accurate description of the short time behavior will be useful for analyzing human performance data, for many applications such as large scale simulations it is only the long time behavior, described by the smallest exponent, that is important. The next section will discuss limiting cases for λ_3.

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2 If one uses the full three-exponent results to define the amplitude e_3 and defines a single exponent approximation to match the e_3 term exactly, then improved results can be obtained in the on-target case:

$$P_D = 1 - e_3 e^{-\lambda_3 t} .$$

For Case D, the discontinuity at t=0 is reduced from 2/3 to 1/3, which will provide a better match to the full result. However, using e_3 to define the single exponent approximation does require the complete evaluation of the three-exponent model, and therefore is less straightforward an approximation than the approximation used in Eq. (III-1). In addition, for observers beginning in the wandering or distracted state, e_3 is typically greater than one so that the approximation is negative at $t = 0$. [For Case D, $e_3 = (4 - 2\,p_0(0))/3$ so that e_3 is greater than one for $p_0(0) < 0.5$]. When Eq. (III-1) breaks down it is best to use the complete model results.

3 Note that the delays calculated here do not include all possible delays. Appendix A discusses the delays associated with the physical process of moving the eye, and possible "orientation" delays at the beginning of search.

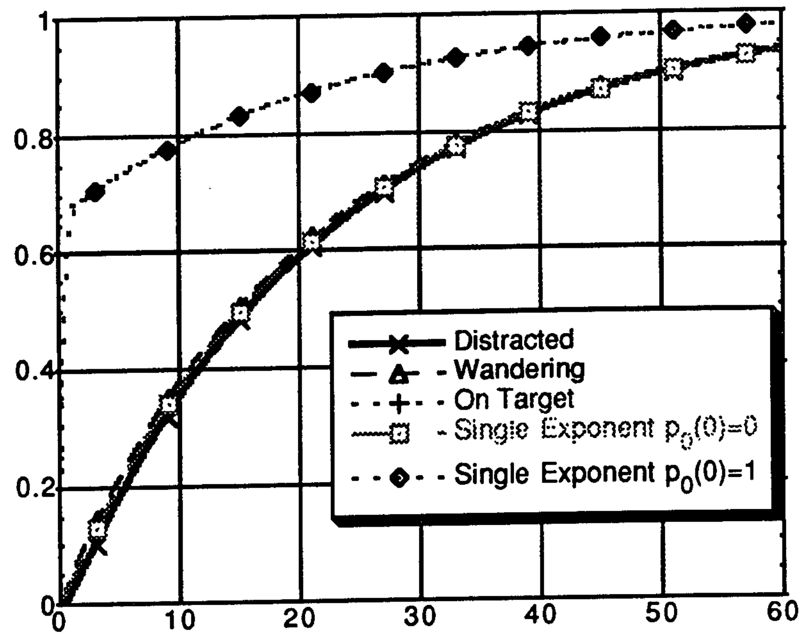

Fig. III-1a. Probability of Detection, S_0 = 0.2

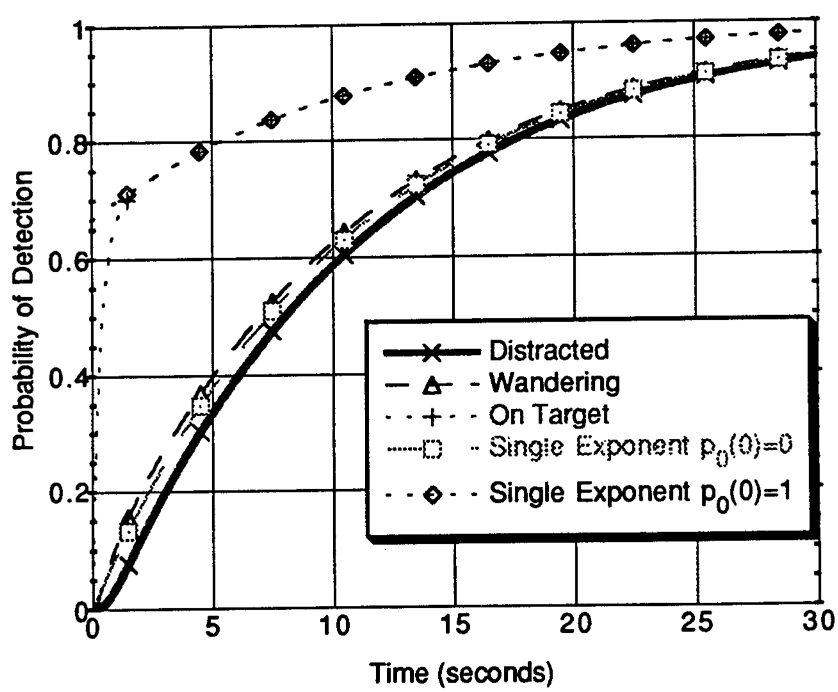

Fig. III-1b. Probability of Detection, S_0 = 0.4

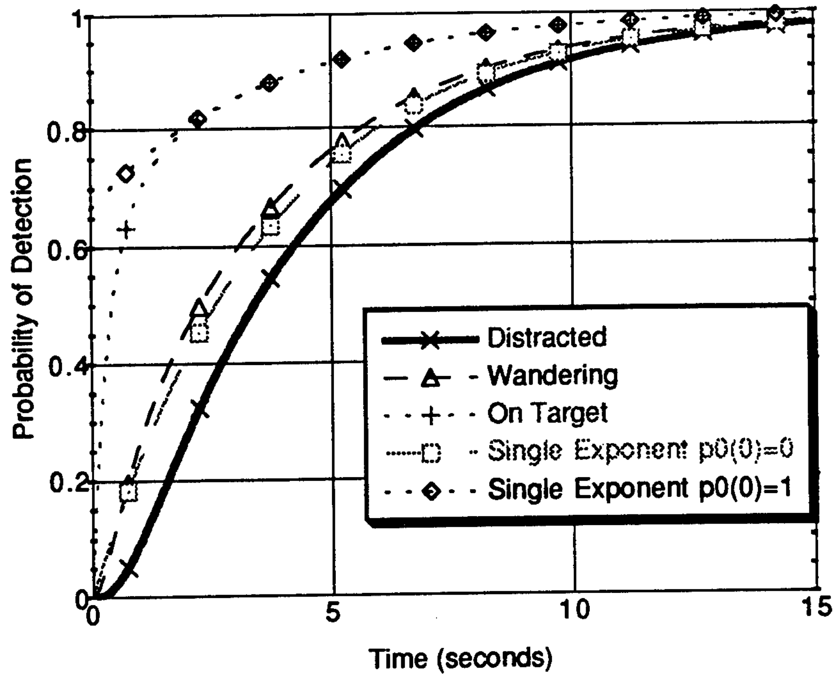

Fig. III-1c. Probability of Detection, S_0 = 1.0

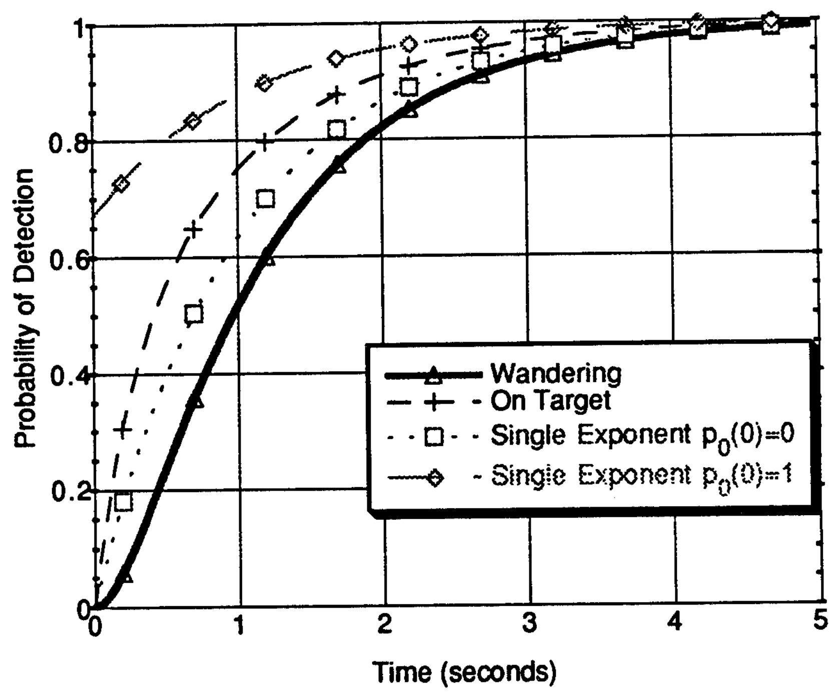

Fig. III-1d. Probability of Detection, S_0 = 2.0

B. CALCULATION OF λ_3

The previous section showed that in many cases the neo-classical model is equivalent to a single exponent model. For these cases the most important contribution of the neo-classical framework is the calculation of the dominant exponent, λ_3. This section will give several special cases where the value of the exponent can be calculated and the factors contributing to it explored.

1. Hard-to-Detect (When Cued) Targets

A limiting case that guarantees the accuracy of the one-exponent limit is that of the hard-to-detect (when cued) target, $\alpha_0 << W$. For this case, λ_3 can be written as[4]

$$\lambda_3 = \alpha_0 \frac{S_0 W + J_0 S}{R(W_0 + J) - (S_0 - J_0)(W_0 - W)} = \alpha_0 p_0^{eq} . \qquad \text{(III-3)}$$

This is equivalent to replacing the random function $\eta(t)$ in the detection equation [Eq. (II-10)] with its average value. For $W = W_0$, and using $J_0 = \kappa S_0$ and $J = \kappa S$ to relate the J and S transition rates we can rewrite Eq. (III-3) as

$$\lambda_3 = \frac{\alpha_0 [S_0 W + J_0 S]}{R(W + J)} = \frac{P_{detIcue} P_{cue}}{T_{sac}} \frac{T_w}{T_{sac}} \frac{1}{(1 + S\, T_w)} , \qquad \text{(III-4)}$$

where we have written $T_w = 1/W$; it is the mean dwell time on the target in the absence of jumping. Note that the jumping terms cancel between the numerator and denominator.

Equation (III-4) has a direct physical interpretation. The effective detection rate is proportional to the cueing and detection probabilities. The rate is increased by a factor of the number of examination fixations per saccade time and reduced by the fraction of time expended on the distracters. The effective detection rate reflects the slowing down of the search process due to competing targets and clutter.

2. Hard-to-Cue, Easy-to-Detect (When Cued) Targets

Another limiting case that guarantees a long search is a target that is hard to cue ($S_0 << 1$). If α_0 is also small, this can be considered to be a subcase of the previous example. On the other hand, for large α_0, the target is detected essentially as soon as it is cued. This corresponds to targets that are easy to detect but possibly hard to be cued to;

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4 This differs from Eq. (II-22) because higher order terms in S_0 and J_0 are included.

once the attention of the observer is drawn to the target, detection is essentially immediate. The probability that the target is attended to initially, $p_0(0)$, is therefore an important initial condition since it determines whether or not there is a chance of rapid detection. For $p_0(0) = 0$, detection will be slower and will depend on the time of first arrival to the target; thus, at least two terms are required.

In the limit of large α_0, $\lambda_2 \approx \alpha_0$, $e_2 \approx p_0(0)$ and the cubic equation for the exponents reduces to a quadratic. The λ_1 and λ_3 exponents and their corresponding amplitudes are given by:

$$\lambda_{1,3} = \frac{R + J_0 \pm \sqrt{(R + J_0)^2 - 4[S_0 W + J_0 S]}}{2} \qquad \text{(III-5a)}$$

$$e_i = \frac{\lambda_j - J_0 p(0) - S_0 w(0) + p_0(0)(\lambda_i - R)}{\lambda_j - \lambda_i} \ . \qquad \text{(III-5b)}$$

If it is also assumed that the target is hard to be cued to (J_0, $S_0 << 1$), then the e_1 amplitude is small and that term can be neglected. The probability of detection is:

$$P_D(t) = p_0(0)\,[\,1 - e^{-\alpha_0 t}\,] + (1 - p_0(0))\,[1 - e^{-\lambda_3 t}\,] + O(J_0, S_0, 1/\alpha_0) \ . \qquad \text{(III-6)}$$

The first term in Eq. (III-6) represents a rapid rise in the probability of detection when cued to the target similar to that illustrated in the numerical examples. The slow exponent (λ_3) in the limit of large α_0 and small cueing rates (J_0, $S_0 << W$) is:

$$\lambda_3 = \frac{S_0 W + J_0 S}{R} = \frac{P_{cue}}{T_{sac}} \frac{(1 + \kappa T_w S)}{(1 + T_w S)} \ . \qquad \text{(III-7)}$$

The first factor is the simple cueing rate to the target; this is reduced by the competing clutter.

3. One Point of Interest

The neoclassical model was developed to handle multiple targets and clutter objects. However, it can be applied to a single-target, minimal-clutter environment; since competing clutter objects are considered analogous to targets, this limit can be described as "one penguin in Antarctica." For this case $S = S_0$, $J = J_0 = 0$, and there is no distinction between W and W_0. In this case $\lambda_2 = W$ and $e_2 = 0$, exactly. The previous expressions

do not reduce to this because it was assumed that $S_0 << S$, which is inappropriate for the single point of interest case. The remaining terms satisfy:

$$\lambda_{1,3} = \frac{W + S_0 + \alpha_0 \pm \sqrt{(W + S_0 + \alpha_0)^2 - 4\alpha_0 S_0}}{2} \qquad \text{(III-8a)}$$

$$e_i = \frac{\lambda_j - \alpha_0 p_0(0)}{\lambda_j - \lambda_i} \quad . \qquad \text{(III-8b)}$$

The single target limit of the neoclassical model is mathematically equivalent to a smoke obscuration model developed earlier: the observer is either looking at the target or he is not. Nicoll and Silk (1991) give a detailed discussion and parametric exploration of the smoke model.

These examples show that the λ_2 exponent and corresponding term are generally associated with balancing multiple points of interest, that is, when competing clutter or multiple targets are modeled. Similarly, the λ_1 exponent and corresponding term are associated with balancing examining points of interest vs. wandering.

4. The Classical Limit: Single, Hard-to-Cue, Easy-to-Detect Target

In the previous example of a single target, two exponents were still required to describe the search and detection process. There are still two distinct processes involved: (1) searching the scene for the target and (2) deciding that it is the target. For small values of α_0, the latter process is slow and may require many revisits to the target before the target is detected. On the other hand, if α_0 is large the target will be detected rapidly and a large number of visits are not required. In this large single target limit the exponents are given by

$$\lambda_1 = \alpha_0 ; \qquad \lambda_3 = S_0 \qquad \text{(III-9a)}$$

$$P_D(t) = p_0(0)\,(1 - e^{-\alpha_0 t}) + (1 - p_0(0))(1 - e^{-S_0 t}) \quad . \qquad \text{(III-9b)}$$

For $\alpha_0 \Rightarrow \infty$, we recover the classical limit given in Eq. (I-7) and can identify S_0 in that case with the reciprocal of the classical time constant: $S_0 = \frac{1}{\tau}$.

C. LARGE-SCALE SIMULATION APPLICATIONS

Although the model provides explicit expressions for the probability of detection as a function of time, certain applications, such as CASTFOREM, approach $P_D(t)$ differently. In these cases it is important to provide the time to detect (if detected). The time to detect is handled in different ways in different large-scale simulations. In Janus Army [Parish and Kellner (1992)], once a P_∞ test is passed (the target could be seen eventually if enough time is permitted) repeated calls are made to a subroutine to calculate $P_D(t)$ to determine the probability that a specific target unit was detected by a specific observer during the preceding time interval. A random number draw then is used to determine if the target is detected or not [that is, a random number r, $0 \le r \le 1$, is compared to $P_D(t)$; if $P_D(t) > r$, the target is declared to be detected]. This process is continued until the target is detected, or the line of sight is broken, or either the target or observer is killed. The use of three exponentials instead of one does not significantly increase the computational burden imposed. Other simulations (e.g., CASTFOREM) make random draws to directly determine the time to detect. For the single exponential form of the classical model, if a random number r is chosen, then the time to detect, t_d, given by

$$t_d = - \tau \ln (1-r) \qquad \text{(III-10)}$$

is a random time-to-detect variable that has the correct probability distribution. Random draws for the time to detect are not quite as simple for the three-exponent neoclassical model as for the single-exponent classical model since a closed form inversion of the $P_D(t)$ equation does not exist. However, no essentially different procedure is required. The necessary inverse of $P_D(t)$ can be performed numerically (using the single exponential approximation as a starting point) or provided in a lookup table, or, where appropriate, the single-exponent approximation to the full neoclassical model can be employed. Since the examples in this section have shown that the single exponent approximation is adequate for almost all applications to field of view search, essentially no change is required in the force-on-force models.

In the next section, extensions of these examples to the case of $P_\infty < 1$ will be discussed, and, in particular, how to compute the time to detect when the search time is limited or the number of visits to the target is limited.

IV. COMPUTING P_∞

The neoclassical framework does not directly provide a mechanism for limiting the long time probability of detection; the separation the detection mechanism from the search process permits a variety of different methods. This section discusses several methods of introducing a $P_\infty < 1$. The first example involves "quitting," that is, abandoning the search process itself. The second subsection discusses changes in the detection process that produce $P_\infty < 1$. This includes: (a) ad hoc methods of P_∞ introduction; (b) models that restrict the number of visits to the target; and (c) nonexponential detection. Appendix D describes a nonstationary restricted visit example.

A. QUITTING THE SEARCH PROCESS

The simplest way to introduce a quitting process is to allow quitting from all the states of the Markov search process. That is, the observer can quit from the wandering state and from the point of interest examination states. If all states quit at a single exponential rate, Q, the Markov equations become

$$\dot{\varepsilon} = -\alpha_0 p_0 - Q\varepsilon \tag{IV-1a}$$

$$\dot{w} = Wp - Sw - Qw + (W_0 - W)p_0 \tag{IV-1b}$$

$$\dot{p}_0 = S_0 w + J_0 p - J p_0 - W_0 p_0 - \alpha_0 p_0 - Q p_0 \ . \tag{IV-1c}$$

This is equivalent to a supervisory process that determines the total search time and moves the observer to a new task with a total search time that is itself exponentially distributed. The solution is simple to state in terms of previous solutions for no quitting, $Q = 0$. The exponents λ_i and amplitudes e_i are computed for $Q = 0$; then, the probability of detection is

$$P_D(t) = \sum_{i=1}^{3} e_i \frac{\lambda_i}{\lambda_i + Q}\left[1 - e^{-(\lambda_i + Q)t}\right] \ . \tag{IV-2}$$

In this formulation, the effects of quitting are easy to see. For example, P_∞ is given by:

$$P_\infty = \sum_{i=1}^{3} e_i \frac{\lambda_i}{\lambda_i + Q} \ . \tag{IV-3a}$$

Evaluating this expression gives the following result:

$$P_\infty = \frac{\Pi + \alpha_0 Q\,[R\ p_0(0) + J_0\, p(0) + S_0 w(0)] + \alpha_0 Q^2 p_0(0)}{\Pi + Q\,[R\hat{W}_0 - \Delta_0 \delta W + \alpha_0 J_0] + Q^2\,[R + \hat{W}_0] + Q^3} \quad . \tag{IV-3b}$$

where $\Pi = \lambda_1\lambda_2\lambda_3 = \alpha_0(S_0W + J_0S)$. For small Q, this is just

$$P_\infty = 1 - Q\langle t_D\rangle + O(Q^2) \quad , \tag{IV-3c}$$

where $\langle t_D\rangle$ is the mean time to detection given in Section II. For situations dominated by the λ_3 term, Eq. (IV-3b) is approximately:

$$P_\infty = \frac{\lambda_3}{\lambda_3 + Q} \quad . \tag{IV-3d}$$

Recall that in the classical model there is a simple relationship presumed to hold between the time constant used and P_∞; recalling the classical search model expressions

$$P_D(t) = P_\infty\,(1 - e^{-t/\tau}) \tag{IV-4a}$$

$$\tau \approx 3.4/P_\infty \qquad P_\infty < 0.9 \tag{IV-4b}$$

$$\tau \approx 6.8\, N_{50} \qquad P_\infty \geq 0.9 \quad . \tag{IV-4c}$$

For the neoclassical model, there is no single time constant. For long times and small values of S_0, the probability of detection is, however, dominated by the slow exponent, λ_3. If we use the single-exponent approximation and simple quitting, we have

$$P_D = \frac{\lambda_3}{\lambda_3 + Q}\,(1 - e^{-(\lambda_3 + Q)t}) \tag{IV-5a}$$

$$\tau = (1 - P_\infty)/Q \quad . \tag{IV-5b}$$

Although Eq. (IV-5b) differs in form from Eq. (IV-4b), it has the same general character (τ decreases with increasing P_∞), and is probably experimentally indistinguishable from the classical relationship due to the enormous range of human variability.

If the observer determines his own quitting time, it might seem more physical to allow quitting only from the wandering state when the observer is not attending to any target candidate. This alternative is algebraically more messy, but might be a more accurate description of quitting. Such an approach is discussed in Appendix B.

B. CHANGING THE DETECTION PROCESS

Instead of changing the search process, the detection process can be changed. The simplest way is to change the detection process to saturate at a $P_\infty < 1$. This may be done by using the ensemble of observers itself to define P_∞, or by restricting the number of visits permitted to the target for the purpose of detection (it is the repeated visits to the target that guarantees $P_\infty = 1$ in the exponential model). Finally, one can consider nonexponential models with built-in limits.

1. Observer Ensemble

One of the interpretations of P_∞ is that it is the fraction of the observer class that will ever see the target. This is easily represented in the neoclassical framework by changing the detection process to include an explicit value of P_∞:

$$\dot{P}_D(t) = \alpha_0 (P_\infty - P_D(t))\, \eta_0(t) \quad . \qquad \text{(IV-6a)}$$

This simply multiplies previous results by P_∞:

$$P_D(t) = P_\infty \sum_{i=1}^{3} e_i \left[1 - e^{-\lambda_i t} \right] \quad . \qquad \text{(IV-6b)}$$

Unlike quitting, this approach does not require any particular relationship between P_∞ and the search model parameters. Any desired relationship can be imposed including the classical relationship to the mean time for search.

2. Restricted Visits

A second method of computing P_∞ from the detection process is to restrict the visits to the target. If only K visits are allowed but each visit is considered to be independent of the others:

$$P_\infty = 1 - (1 - P_{visit})^K \quad . \qquad \text{(IV-7)}$$

Of course, since the search process is decoupled from the detection process in the neoclassical model, the observer may still fixate on the target and dwell on it, but without increasing the detection probability. This is not simply an artifact of the model; the fixations of observers often return to objects about which a decision has already been made.

The procedure for computing the probability of detection for a restricted visit detection model is as follows:

(1) Begin at the initial conditions, compute time to first visit.

(2) Compute increment of probability during visit.

(3) Return to the search.

(4) Repeat from step (1) until the effect of K visits is included.

The complete distribution of arrival times is given by $\alpha_0 \Rightarrow \infty$ limit; in this case, $e_2 = p_0(0)$ exactly and $\lambda_2 = \alpha_0 \Rightarrow \infty$ [see also Eq. (II-20)]. The remaining amplitudes and exponents were given in Eq. (III-5) and are repeated here for convenience:

$$\lambda_{1,3} = \frac{R + J_0 \pm \sqrt{(R + J_0)^2 - 4[S_0 W + J_0 S]}}{2} \qquad \text{(IV-8a)}$$

$$e_i = \frac{\lambda_j - J_0 p(0) - S_0 w(0) + p_0(0)(\lambda_i - R)}{\lambda_j - \lambda_i} \quad . \qquad \text{(IV-8b)}$$

The cumulative probability distribution of arrival times starting from any initial condition is therefore:

$$P_{arrival}(t) = p_0(0) + \sum_{i=1,3} e_i \left(1 - e^{-\lambda_i t}\right) \quad . \qquad \text{(IV-9)}$$

Equation (IV-9) does not involve the issues of whether or not W_0 is taken equal to W; only the value of W determines the arrival time at the target.[1]

Each of the cases of K visits is straightforward to compute analytically. As an example, the case for a single visit is given by

$$P_D(t) = p_0(0) \frac{\alpha_0}{W_0 + J - J_0 + \alpha_0} \left(1 - e^{-(W_0 + J - J_0 + \alpha_0)t}\right)$$

$$+ \sum_{i=1,3} \frac{\alpha_0 e_i \lambda_i}{W_0 + J - J_0 + \alpha_0 - \lambda_i} \left[\frac{(1 - e^{-\lambda_i t})}{\lambda_i} - \frac{(1 - e^{-(W_0 + J - J_0 + \alpha_0)t})}{W_0 + J - J_0 + \alpha_0}\right] \qquad \text{(IV-10a)}$$

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1 For a single point of interest, $\lambda_1 = W$ and $\lambda_2 = S_0$ exactly. $e_1 = 0$ and $e_3 = 1 - p_0(0)$. Only a single exponent is needed to describe the arrival time, S_0. The extra complexity of two exponents in the general arrival time expression allows for the effects of competing targets and clutter.

$$P_\infty = \frac{\alpha_0}{W_0 + J - J_0 + \alpha_0} = P_{visit} \ . \qquad \text{(IV-10b)}$$

Calculations for subsequent visits are simplified by the fact the boundary conditions are fixed and simple:

$$w(0) = \frac{W_0}{W_0 + J - J_0} \qquad \text{(IV-11a)}$$

$$p(0) = \frac{J - J_0}{W_0 + J - J_0} \qquad \text{(IV-11b)}$$

$$p_0(0) = 0 \ . \qquad \text{(IV-11c)}$$

The algebra begins to be a little cumbersome for a large number of visits but the K = 2 case is worked out in detail in Appendix D for a nonstationary version of the neoclassical model.

If a complete analytic solution is not desired for the K visit problem, a Monte Carlo approach can be taken; this might be appropriate for large scale simulations that use the time to detect as the basic methodology.[2] Two "clocks" will be used to describe the search and detection processes. The first clock is a running clock that provides the final time to detect; the second clock keeps track of the time spent attending to the target. Draw a random number r ($0 \leq r \leq 1$) to represent the probability of detection.

(1) Begin at the initial conditions, draw a random time from the arrival time distribution to first visit. Add this to the running time-to-detect clock.

(2) Draw a random time for the duration of the visit from an exponential distribution with mean time $1/(W_0 + J - J_0)$ and add to the time on the time-to-detect clock and to the time-on-target clock. Compute increment of probability during visit. If the accumulated probability of detection exceeds r, use the current time-to-detect clock reading and exit.

(3) Return to search in a state with conditions set by Eq. (IV-11).

(4) Repeat from step (1) until the effect of K visits is included. If K visits occur without achieving detection, the time to detect is infinite (never detected).

It is easy to see that this reproduces the results obtained for the analytic solution. The analytic results are, when available, simpler to use.

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2 Suggested by a remark of G. Witus for a single point of interest model.

3. Nonexponential Detection Processes

As discussed above, any detection process can be used in the model, including nonexponential processes. The simplest extension is to use models that depend on the time on target, T(t), since these can all be calculated from the exponential model, as will now be shown. The general solution for the exponential detection model,

$$P_D(t) = \left< 1 - e^{-\alpha_0 \int_0^t \eta_0(s)\, ds} \right> \qquad \text{(IV-12)}$$

is the "generating" function of the moments of the "time on target function," T. Consider successive derivatives of Eq. (IV-12) evaluated at $\alpha_0 = 0$. Each derivative brings down another power of the time on target integral yielding

$$(-1)^{n+1} \frac{\partial^n P_D(\alpha_0,t)}{\partial \alpha_0^n} \Big|_{\alpha_0 = 0} = \left< T^n \right> \quad . \qquad \text{(IV-13)}$$

This generating function property of the exponential solution is analogous to the approach taken in field theory to evaluate the correlation functions of the theory [Chang et al. (1992)]. Any model of detection that is a function of the time on target alone, D(T), can be computed from the exponential model. For example, consider

$$D(T) = T/(1+T) \qquad \text{(IV-14a)}$$

$$P_D(t) = \left< D(T) \right> = \left< \frac{T}{1+T} \right> = \sum_{n=1}^{\infty} (-1)^{n-1} \left< T^n \right> \quad . \qquad \text{(IV-14b)}$$

The generating function approach defines all detection models that depend only on the time on target provided by the search. The simplest of these models assumes that the probability of detection is linear in the time on target, with a proportional constant, β:

$$P_D = \beta \left< T \right> \quad . \qquad \text{(IV-15a)}$$

This allows multiple detections on single target; therefore, this model is used primarily for illustrative purposes. Calculating the average time on target, one has

$$\langle T(t)\rangle = \frac{p_0^{eq}(1 - e^{-Q t})}{Q}$$

$$+ \frac{\{J_0[p^{eq} - p(0)] + S_0[w^{eq} - w(0)]\}(1 - e^{-(R + Q)t})}{(S - J)(R + Q)} \qquad \text{(IV-15b)}$$

$$- \frac{\{J_0[p^{eq} - p(0)] + S_0[w^{eq} - w(0)] + (S - J)\,[p_0^{eq} - p_0(0)\,]\}(1 - e^{-(W + J + Q)t})}{(S - J)(W + J + Q)}$$

$$p_0^{eq} = \frac{S_0 W + J_0 S}{R\,(W + J)}, \quad p^{eq} = S\,/\,R, \quad w^{eq} = W\,/R \ . \qquad \text{(IV-15c)}$$

In Eq. (IV-15), simple quitting and $W_0 = W$ have been assumed. The second and third terms vanish for equilibrium initial conditions.

Eq. (IV-15) does not provide a particularly interesting nonexponential model since it does not provide a value of P_∞ without the aid of a quitting mechanism. Another possible model would be to take P_D to be given by

$$P_D(t) = \left\langle \frac{T(t)}{1 + \frac{T(t)}{P_\infty}} \right\rangle \qquad \text{(IV-16)}$$

and use Eq. (IV-13) to calculate the required moments.

Witus (1993) has suggested another model that produces a saturated P_∞ by assuming that the glimpses of the target are not independent, as is assumed in the exponential model, but gradually become less effective due to correlation of information between the glimpses. The efficiency of each glimpse depends only on the number of preceding glimpses so this model can be included in the neoclassical framework by using a nonstationary approach. An analogous model could be represented by a detection process equation of the form

$$\dot{P}_D = \alpha_0(T(t))\,(1 - P_D(t))\,\eta_0(t) \ . \qquad \text{(IV-17)}$$

That is, the value of α_0 depends on the time on target; this leads to a nonstationary Markov process. In the nonstationary example given in Appendix D, α_0 has different values for each visit to the target, but does not vary during a visit. More general cases can be handled, however, in a similar fashion, or a Monte Carlo approach can be applied.

V. MULTIPLE-REGION, FIELD-OF-REGARD SEARCH

A. MORE THAN ONE SEARCH REGION

All of the above discussion has used a single region or field-of-view perspective. A single region is implicit in the use of averaged transition rates; for example, the use of a single wandering state with a single transition rate, S_i, to the ith point of interest rather than transition rates that depend on the target eccentricity. This approximation is not suitable for field-of-regard search or for multiple regions of distinctive clutter within a single field of view. For example, the search of a field of view containing a background tree line and foreground open meadow is better described as a multiple region search since the search strategies for the two regions are likely to be different. This section extends the neoclassical framework to multiple-region or field-of-regard search.

A guide to the needed approach is to consider the processes that were modeled in the neoclassical representation of field of view search. The single region search exhibited competition between the various processes modeled only for short times. For all but the shortest searches (Cases A, B, C in Section III) the competition between processes is confined to the first few target dwells. There are typically two fast exponents, decaying within a few target examination times, and one slow exponent, describing the overall search. Although the initial conditions are important for the first few seconds, the slow exponent describes the search well. For the shortest searches (Case D in Section III), all three exponents are fast, corresponding to a few seconds at most.

In no case is there a competition between two slow exponents. This absence of a slow-slow competition is a consequence of the approximations made to describe the search process; in particular, it is a result of assuming an average transition rate independent of the distance travelled by the eye from one fixation to another (the target eccentricity). Consider for illustrative purposes a scene consisting of two separate regions. Suppose that the observer searches each of these regions diligently and rarely crosses the boundary from one region to another (see Fig. V-1). This search will have two slow exponents:

(1) The complete in-FOV search process

(2) Jumping between different FOV.

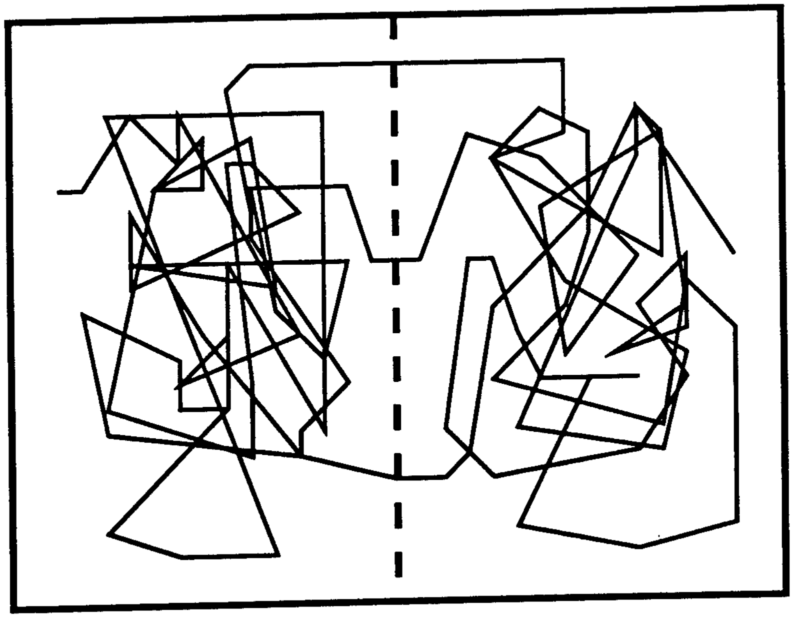

Figure V-1. A Notional Multiregion Search

This corresponds to distinguishing two different transition rates: one for large separations between the observer line of sight and a target (the transitions between regions) and another for small separations (the transitions within one region). We may imagine partitioning the scene into many different regions with different transition rates between regions depending on their relative separations. This would presumably lead to a number of slow exponents: one basic slow exponent for the in-region search and a range of slow exponents describing the transition between regions. Therefore, the presence of only one slow exponent in the neoclassical model is a direct consequence of the approximation of using a single transition rate averaged over a single region. For field-of-regard search or for multiple clutter domain regions, a single transition rate is inadequate to address the phenomenology.

This section discusses several approaches to addressing this deficiency in the neoclassical model. The point of comparison in all cases is a Markov description of the search process that permits the specification of separate transition rates from each point of the total field of regard to every other point. Each of the approaches will be an approximation to that model. The most detailed, but "parameter rich," mathematical approach leads to a system with 2M + 1 exponents where M is the number of regions or fields of view into which the total field of regard is divided. For small M, this is not an insuperable burden for numerical calculations and modeling; however, it increases the number of parameters to be estimated or modeled. A second approach makes additional approximations and reduces the number of states to provide a five-exponent system for any

field of regard. Finally, the third approach depends on the fact that for searches confined to a single search region the neoclassical field of view model developed in Section II applies and is dominated (except for very short times) by the slow exponent. By approximating the search within a single region as a single exponent, the interaction between regions can be approximated by a two-exponent model for which both of the exponents are slow. The last approach is recommended for its simplicity and directness; the first two have great flexibility at the cost of increased algebraic clutter.

B. MATHEMATICAL FRAMEWORK

1. Parameter-Rich Multiregion Model

The first approach uses the largest number of adjustable parameters and exponents of the multiple-region approaches considered, but offers the potential for the greatest flexibility in analyzing actual performance data. It also provides the basis for the simpler models by defining the general framework of the multiregion problem. In order to represent this effect systematically, the field of regard can be divided into a number, say M, of smaller regions, as illustrated in Fig. V-2.

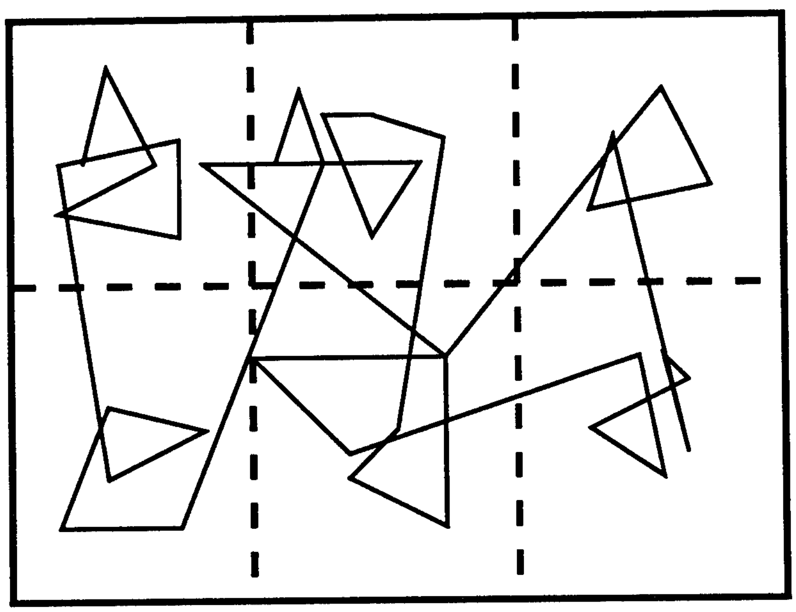

Figure V-2. A Field-of-Regard Search Divided Into Field of View

These regions can be distinct fields of view or different clutter regions within a field of view. The occupation probabilities for the different regions are defined as

$w_{(k)}$ = probability of wandering in the kth region (V-1a)

$p_{(k),i}$ = probability of being fixated on the ith target candidate in the kth region (V-1b)

$p_{(k)}$ = probability of being fixated on some target candidate in the kth region (V-1c)

$$w = \sum_j w_{(j)} \quad \text{= probability that the observer is wandering somewhere} \tag{V-1d}$$

$$p = \sum_j p_{(j)} \quad \text{= probability that the observer is fixated somewhere} \tag{V-1e}$$

$$\varepsilon \quad = \quad w + p = \quad \text{total probability} \tag{V-1f}$$

in the absence of an absorbing state $\varepsilon = 1$. Subscripts in parenthesis [e.g., (k), (j)] denote different regions; objects within a region are denoted by plain subscripts (e.g., i). In the spirit of the neoclassical model, it is assumed that the transition rates to a state in the kth region from a state in the jth region are represented by an average value. To define

$$S^{(j)}_{(k),i} \quad = \quad \text{Rate into ith POI of kth region from wandering in jth region} \tag{V-2a}$$

$$J^{(j)}_{(k),i} \quad = \quad \text{Rate into ith POI of kth region from a POI in jth region} \tag{V-2b}$$

$$W^{(j)}_{(k)} \quad = \quad \text{Rate into wandering in kth region from a POI in jth region} \tag{V-2c}$$

$$L^{(j)}_{(k)} \quad = \quad \text{Rate into wandering in kth region from wandering in jth region} \ . \tag{V-2d}$$

All of these rates are in principle measurable in a search experiment and could be computed by an appropriate psychophysical model from target and background characteristics. Define the regional rates by summing over the targets in a region:

$$S^{(j)}_{(k)} = \sum_i S^{(j)}_{(k),i} \qquad\qquad J^{(j)}_{(k)} = \sum_i J^{(j)}_{(k),i} \ . \tag{V-2e}$$

Armed with these transition rates the Markov equations for search are:

$$\dot{p}_{(k),i} = \sum_{(j)} [\, S^{(j)}_{(k),i}\, w_{(j)} + J^{(j)}_{(k),i}\, p_{(j)} - W^{(k)}_{(j)}\, p_{(k),i} - J^{(k)}_{(j)}\, p_{(k),i} \,] \tag{V-3a}$$

$$\dot{w}_{(k)} = \sum_j [\, W^{(j)}_{(k)}\, p_{(j)} + L^{(j)}_{(k)}\, w_{(j)} - S^{(k)}_{(j)}\, w_{(k)} - L^{(k)}_{(j)}\, w_{(k)} \,] \tag{V-3b}$$

$$\dot{p}_{(k)} = \sum_j [\, S^{(j)}_{(k)}\, w_{(j)} + J^{(j)}_{(k)}\, p_{(j)} - W^{(k)}_{(j)}\, p_{(k)} - J^{(k)}_{(j)}\, p_{(k)} \,] \ . \tag{V-3c}$$

Note that the equations defining the overall wandering and examining probabilities in the regions (Eqs. V-3b and V-3c) depend only on the average transition rates. Detection of the 0th POI in the 0th region is modeled by adding an absorbing state; Eq. (V-3a) is modified for that POI:

$$\dot{p}_{(0),0} = \sum_{(j)} [S^{(j)}_{(0),0} w_{(j)} + J^{(j)}_{(0),0} p_{(j)} - W^{(0)}_{(j)} p_{(0),0} - J^{(0)}_{(j)} p_{(0),0}] - \alpha_{(0),0} p_{(0),0} \quad . \tag{V-4a}$$

The overall probability equation is unchanged in form:

$$\dot{\varepsilon} = - \alpha_{(0),0} \, p_{(0),0} \quad . \tag{V-4b}$$

Eqs. (V-4a) and (V-4b) can be combined with the $w_{(0)}$ equation and the 2(M-1) equations for $w_{(k)}$ and $p_{(k)}$ ($k \neq 0$) to give a system of 2M + 1 equations. The 2M + 1 eigenvalues are determined from a polynomial of degree 2M + 1 and 2M + 1 initial conditions must be specified. We would expect 2 fast exponents describing rapid processes of the order of a few seconds or less (the analogs of λ_1 and λ_2), one slow exponent proportional to α_0 describing detection (analogous to λ_3) and 2M-2 slow to fast exponents describing the relationships between the regions.

For numerical work and a relatively small number of regions ($M \approx 5$ or less) this is not insuperably complicated. More difficult is the specification or measurement of all the transition rates and initial conditions. The number of adjustable parameters can be large for large M. There are M^2 values of $W_{(k)}{}^{(j)}$, $S_{(k)}{}^{(j)}$, and $J_{(k)}{}^{(j)}$ that must be specified as averages over the targets in a region; and, in addition, rates for each of the targets must be defined. All of the rates have in principle well-defined values in terms of measurable transition rates but the large number of rates will limit the accuracy of their determination in any practical experiment. However, an examination of actual human performance data or by the use of modeling might show that many of these rates could be neglected. This remains to be determined by further experiments or modeling efforts. However, even if this model were developed in detail, the interpretation of the results would likely be more complicated than desirable for purposes of modeling overall patterns of search. Further approximations are needed.

2. Parameter-Sparse Multiregion Model

The parameter-rich model has a large number of parameters because it permits the transition rates from each region to every other region to be separately specified. Simplifications can be made by averaging the rates over the different regions. This can be

done in a number of different ways, giving rise to different approximations. A particularly simple approximation is to average all of the inter-region transition rates so that all the rates can be divided into internal and external components. That is, there is a single transition rate for all transitions of the same type from one region to another.

$$S^{(j)}_{(k),\,i} = \delta_{kj}\, S_{(k),i} + (1 - \delta_{kj})\, S^{ext}_{(k),i} \tag{V-5a}$$

$$J^{(j)}_{(k),\,i} = \delta_{kj}\, J_{(k),i} + (1 - \delta_{kj})\, J^{ext}_{(k),i} \tag{V-5b}$$

$$W^{(j)}_{(k)} = \delta_{kj}\, W_{(k)} + (1 - \delta_{kj})\, W^{ext}_{(k)} \quad . \tag{V-5c}$$

We also assume that the linking transitions between the wandering states depend only on the destination.

$$L^{(j)}_{(k)} = L_{(k)} \quad . \tag{V-5d}$$

It is then convenient to define the total linking rate, L:

$$L = \sum_{(k)} L_{(k)} \quad . \tag{V-6a}$$

Finally, it is assumed that the average rates of moving from any of the wandering states to a point of interest, moving from a point of interest to any of the wandering states, or moving from a point of interest to any another point of interest are independent of the region of origin. Thus the corresponding total transition rates are independent of the region of origin:

$$S = \sum_{(k)} S^{(j)}_{(k)} ; \quad W = \sum_{(k)} W^{(j)}_{(k)} ; \quad J = \sum_{(k)} J^{(j)}_{(k)} \quad . \tag{V-6b}$$

Defining the total transition rates from states in one region to states in a different region,

$$S^{ext} = \sum_{(k)} S^{ext}_{(k)} ; \; J^{ext} = \sum_{(k)} J^{ext}_{(k)} ; \; W^{ext} = \sum_{(k)} W^{ext}_{(k)} \quad . \tag{V-6c}$$

Then combining the definitions in Eq. (V-5) and Eq. (V-6) one has

$$S_{(j)} - S^{ext}_{(j)} = S - S^{ext}$$

$$J_{(j)} - J^{ext}_{(j)} = J - J^{ext} \qquad \text{(V-7)}$$

$$W_{(j)} - W^{ext}_{(j)} = W - W^{ext} \quad .$$

The difference between the internal transition rate and the external transition rate into any region is independent of the region. With all of this notational apparatus, and defining $w_{ext} = w - w_{(0)}$, and $p_{ext} = p - p_{(0)}$, the parameter-rich model equations reduce to five equations:

$$\dot{\varepsilon} = -\alpha_{(0),0}\, p_{(0),0} \qquad \text{(V-7a)}$$

$$\dot{w} = -S\, w + W\, p \qquad \text{(V-7b)}$$

$$\dot{p}_{(0),0} = S_{(0),0}\, w + J_{(0),0}(\varepsilon - w) - [W + J + \alpha_{(0),0}]\, p_{(0),0} + (S^{ext}_{(0),0} - S_{(0),0})\, w_{ext} + (J^{ext}_{(0),0} - J_{(0),0})\, p_{ext} \qquad \text{(V-7c)}$$

$$\dot{w}_{ext} = (W - W^{ext})p_{ext} + (W^{ext} - W^{ext}_{(0)})p + (L - L_{(0)})\, w - (S + L)\, w_{ext} \qquad \text{(V-7d)}$$

$$\dot{p}_{ext} = (S - S^{ext})w_{ext} + (S^{ext} - S^{ext}_{(0)})w + (J^{ext} - J^{ext}_{(0)})\, p - (W + J^{ext})p_{ext} \quad . \qquad \text{(V-7e)}$$

These five equations are solved in the same manner as used in the three-state neoclassical models.[1] They lead to a set of 5 exponents with two slow exponents and three fast. Although a complete discussion of the 5 exponents requires the solution of a quintic equation, some insight can be gained by considering the equations for $\alpha_{(0),0} = 0$. Then the first three exponents are as in the single region model

$$\lambda_1 = W + S; \qquad \lambda_2 = W + J; \qquad \lambda_3 = 0 \quad . \qquad \text{(V-8a)}$$

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1 Other approximations for special cases such as a two-region model are also possible. A two-region model leads to a slightly more general five-exponent representation.

The addition exponents satisfy the quadratic equation:

$$\lambda^2 - (S + W + L + J^{ext})\lambda + L(W + J^{ext}) + S(W^{ext} + J^{ext}) + S^{ext}(W - W^{ext}) = 0 \quad . \tag{V-8b}$$

If the terms connecting regions are assumed to be small, L, J^{ext}, W^{ext}, $S^{ext} << W$, the solutions to the quadratic are approximately

$$\lambda_4 = W + S; \qquad \lambda_5 = \frac{L W + S(W^{ext} + J^{ext}) + S^{ext} W}{S + W} \quad . \tag{V-8c}$$

Thus, λ_3 and λ_5 are the slow exponents and will dominate the search process while the other fast exponents decay rapidly as in the previously studied cases.

Given a sufficiently accurate psychophysical model for predicting the rates and sufficiently detailed experiments to validate the model this approach may be useful. However, the large number of parameters even in this sparse parameter approximation still makes it hard to visualize the significance of each individual parameter. Since this model is dominated by two slow exponents it would be better to have a model that directly addressed the two dominant processes.

3. Hierarchical Approximation to Multiregion Search

Although the methods described above provide a mathematically cogent approach to modeling multiregion search, the number of adjustable parameters is too large for a general purpose search model. The simplest method of addressing the multiregion depends upon an analogy drawn between Fig. V-1, which describes multiregion search, and Fig. II-1, which describes search in a single region. The basic idea is to treat each region (or field of view) as a point of interest in a large-scale search of the entire search region (field of regard). This coarse search is coupled with a single-exponent approximation of the search within the region to give a complete description of the process. This results in a simple, two-slow-exponent approximation to the search.

As discussed in Section II, the probability of detection is an average over all possible search paths of the simple exponential of the time on target:

$$P_D(t) = \left< 1 - e^{-\alpha_0 T(t)} \right>_{\text{Search Paths}} \quad . \tag{V-9a}$$

For multiregion search the search paths can be divided into path segments within a single region (FOV) containing the target and segments that extend over other regions (FOR); obviously, while the path is in a different region, no time is accumulated on target.

Conceptually, we can factor the search paths into FOV and FOR portions. The probability of detection can be written as:

$$P_D(t) = \langle \langle 1 - e^{-\alpha_0 T(t)} \rangle_{\text{FOV paths}} \rangle_{\text{FOR paths}} \quad . \tag{V-9b}$$

This in itself does not simplify the problem much. However, the study of the neoclassical model results in the previous sections shows that in many cases the neoclassical model can be approximated by a single exponential except at short times. This short time behavior is less likely to be of interest in a total search field model. Therefore, Eq. (V-7b) can be simplified if the sum over the single-region paths is replaced with a single effective exponential:

$$P_D(t) = \langle 1 - e^{-\lambda_3 t} \rangle_{\text{FOR paths}} \quad . \tag{V-9c}$$

This is, of course, the starting point of the neoclassical model with λ_3 replacing α_0. The previously developed tools can be immediately applied. Consonant with the neoclassical approach, it is assumed that the search among different regions is described by a Markov process with transition rates that are averages over the larger search field; the transition rate into the ith region is denoted U_i. The model is simpler than the three-exponent neoclassical description since there is no natural analog for the wandering state: the observer is always at one point of interest (= region = field of view) or another. Thus the analogies are:

p_i (fixated on the ith target) $\Leftrightarrow$ $p^{(i)}$ (searching in the ith region)

α_0 (detecting target if fixated) $\Leftrightarrow$ λ_3 (completing single region search)

J_k (jumping from POI to kth POI) $\Leftrightarrow$ U_k = (average transition rate into the kth region)

$J = \sum_j J_j$ (total jumping rate) $\Leftrightarrow$ $U = \sum_j U_j$ (total rate for field of regard search)

and for the multiregion case: $S_i = S = W = 0$. For this hierarchical view, there are two exponents characterizing the total search, λ_a and λ_b. For a target in the 0th region, and whose slow neoclassical exponent is λ_3, the two exponents describing the total search are:

$$\lambda_{a,b} = \frac{U + \lambda_3 \pm \sqrt{(U + \lambda_3)^2 - 4U_0\lambda_3}}{2} \quad . \tag{V-10a}$$

The probability of detection is:

$$P_D(t) = e_a (1 - e^{-\lambda_a t}) + e_b (1 - e^{-\lambda_b t}) \qquad \text{(V-10b)}$$

where the amplitudes are

$$e_i = \frac{\lambda_j - \lambda_3 \, p_{(0)}(0)}{\lambda_j - \lambda_i} \qquad \text{(V-10c)}$$

and $p_{(0)}(0)$ is the probability that the observer starts the search in the correct search region.

Figure V-3 shows a sample result for 2 fields of view, with $\lambda_3 = 1/10$, $U_0 = 1/20$, $U = 1/10$ and several initial conditions. In this case, the exponents are $(1 \pm \sqrt{2}/2)/10 \approx \{0.17, 0.03\}$. These exponents are both slow but are relatively widely separated; the slower of the two exponents suggests an overall search time constant of the order of 34 seconds. The multiregion result depends on the initial conditions of search—in this case, whether or not the observer starts in the correct field of view. Note that there are significant deviations from the single-exponent model on the scale of the total search.

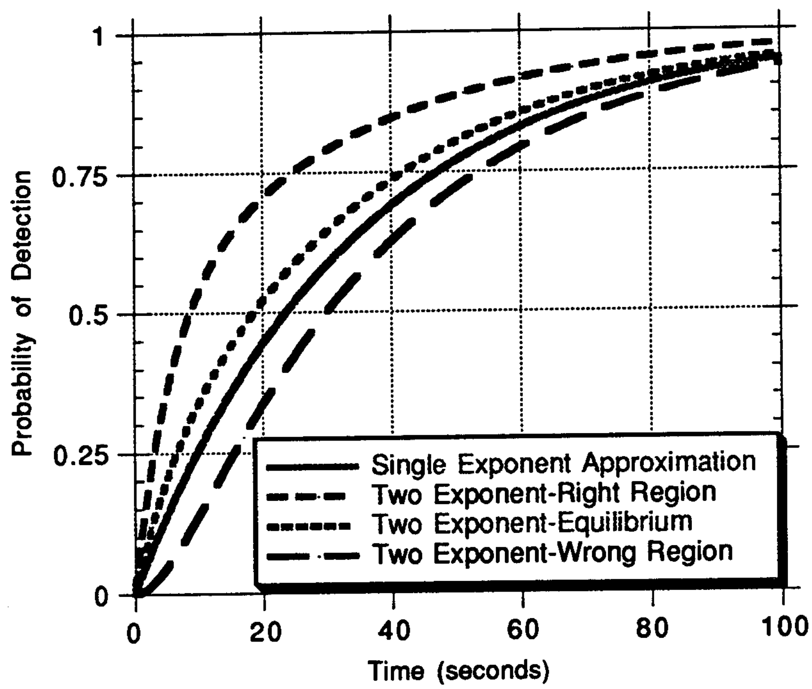

Figure V-3. Multiregion Search Timelines

Other expressions analogous to those of the single-region neoclassical model can be obtained. For example, the mean time to detect in the multiregion case is:

$$\langle t_D \rangle = \frac{1}{\lambda_3}\frac{U}{U_0} + \frac{1}{U_0}[1 - p_{(0)}(0)] \quad . \qquad \text{(V-11)}$$

For the numerical example given in Fig. V-3, the predicted mean time to detect for the different initial conditions varies from 20 to 40 seconds with a mean value of 30 seconds for equilibrium starting conditions. If there are M regions which are equally attractive, then $U/U_0 \approx$ M and Eq. (V-11) becomes

$$\langle t_D \rangle \approx \frac{M}{\lambda_3} + \frac{M}{U}[1 - p_{(0)}(0)] \quad . \qquad \text{(V-12)}$$

If $1/\lambda_3$ is interpreted as the mean time to search a typical region, then the first term in Eq. (V-12) represents the mean time to search M regions. The remaining term adds to the time to detect and corresponds to the inefficiencies in the process. That is, some regions will be over-searched. Similar remarks apply to the three-exponent results but are easier to see in Eq. (V-12) since the number of terms is reduced.[2]

The size of U_0 can be estimated by computing the mean probability of finding the target on the first visit to the appropriate region:

$$P_{visit} = \frac{\lambda_3}{\lambda_3 + U - U_0} \quad . \qquad \text{(V-13)}$$

For the numerical example used in Fig. V-3, the probability of finding the target on a single examination of the region containing the target is 2/3. If the observer does a relatively complete search of each region before moving on to the next, then U will be on the order of λ_3. The precise value will depend on more detailed modeling of the psychophysical processes or experimental confirmation.

Other analogies to the single-region search model can also be drawn. P_∞ can be introduced into the model at either the single region or the multiregion level. Further experimental data on the mechanisms for P_∞ will be required before a choice is made. One simple approach is to calculate for the multiregion model the result of assuming that each region is searched only once (but targets can be revisited within the region); in this case P_∞

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2 It is now easy to see that this relationship is partly a consequence of assuming that the detection does not alter the search itself. If the observer knew how many targets were in a particular region, then once they were all found, he could avoid revisiting the region. The use of the Markov process, which permits revisits (to a target in the single-region case, and to a region in the multiregion case) introduces re-visit inefficiencies.

is given by P_{visit} [Eq. (V-13)]. The single visit result is easily calculated in the multi-region hierarchical model—not even a quadratic has to be solved:

$$P_D(t) = p_{(0)}(0)\frac{\lambda_3}{\lambda_3 + U - U_0}(1 - e^{-(U - U_0 + \lambda_3)t})$$

$$+ (1 - p_{(0)}(0))\frac{U_0\lambda_3}{\lambda_3 + U - 2U_0}\left[\frac{1 - e^{-U_0 t}}{U_0} - \frac{1 - e^{-(U - U_0 + \lambda_3)t}}{U - U_0 + \lambda_3}\right] . \qquad \text{(V-14)}$$

Continuing the numerical example, Fig. V-4 compares the result of Eq. (V-14) to the previous results shown in Fig. V-3 for unlimited search. The effect of the initial conditions is even more pronounced. Note that the short time behavior (< 20 seconds) is quite similar to the multiple revisit case.

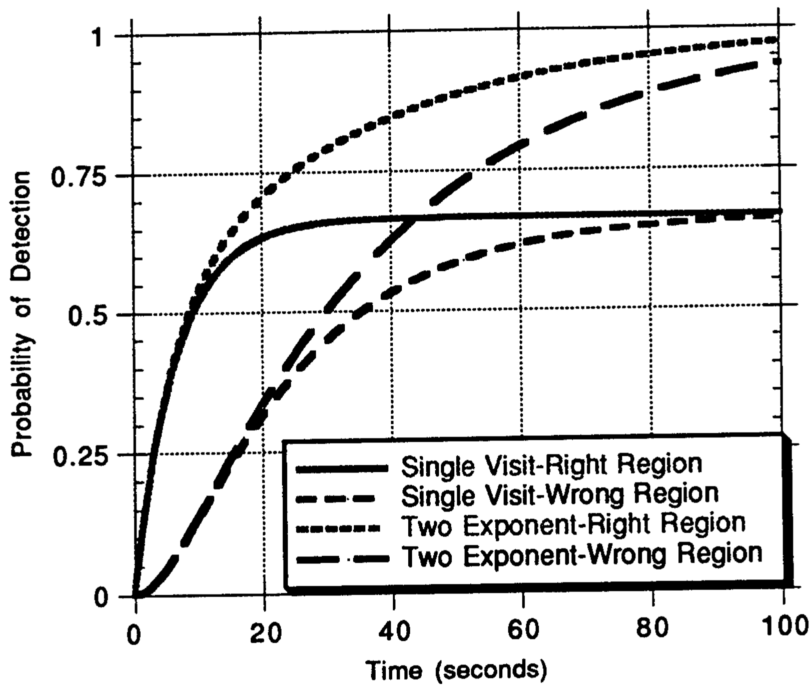

Figure V-4. Single Visit Multiregion Search

VI. DISCUSSION

This paper has introduced a framework for describing search processes within a Markov methodology. Relatively straightforward models can be developed for single-region (field-of-view) and multiregion (field-of-regard) search.

The proposed neoclassical search model for single-region search is a tractable mathematical model with useful properties. The model includes three basic processes: examining the target of interest, examining other target candidates or clutter, and wandering around the scene as a whole. The model describes:

- Multiple targets. Each target is considered separately with the remaining targets acting as distracters.
- False alarms. Clutter objects and targets are described in a single uniform formalism permitting parallel treatments of false and correct detections.
- Clutter. The clutter in a scene can act both as a distraction and as a source of false detections.
- Different models of detection. Different models of the detection process can be used within the same search paradigm.
- Different "initial conditions" of search. These can affect the predicted probability of detection, particularly for short times.
- Different models for P_{∞}. Almost any detection model can be accommodated.

The model includes parameters to describe the cueing and detection processes, which can be computed from models of the human visual process, estimated from the current models or directly measured in eye fixation experiments. All the parameters of the model correspond to experimentally measurable transition rates so that the assumptions and parameters of the model can be validated in detail.

For single-region search, the model replaces the single-exponent classical search model with three exponentials. This is a relatively minor increase in complexity and the model could be straightforwardly included in large-scale simulations such as Janus and CASTFOREM in a single-exponent approximation or in the full three-exponent form. The three exponents are typically divided into one slow exponent that describes the overall probability of detection at moderate to long times and two fast exponents that are important

only for short times (less than a few seconds) As the examples discussed show, the model has considerable scope for describing details of the search process found in human performance experiments. However, the primary value of the model is to provide a more fundamental description of the dominant slow exponent.

For multiregion, or field-of-regard, search, a number of alternative models of differing algebraic complexity were considered. Even the most complex of these is not computationally intensive and could be used as part of a combat simulation model. However, these alternatives would require the specification of a large number of parameters that, although theoretically calculable in a sufficiently powerful psychophysical vision model, would probably prove to be impractical for everyday use. A hierarchical model of multiregion search is proposed as a minimal parameter model. For this approach, one uses a single-exponent approximation for search within each individual region and a coarse-grained model to describe search among the different regions. In this case, the processes modeled are searching the region containing the target, and searching other possible regions (there is no analog to the wandering process). This provides a two-exponent model of multiregion search. These exponents are expected to be more nearly comparable in size; both exponents are important for all times, long and short.

The basic approach can be applied to a number of other search problems not considered here. For example, consider the problem of varying the size of the field of view. An observer can examine the search area in the wide field of view, switching to narrow field of view when attracted by a possible target. This could be represented in a manner similar to the proposed hierarchical multiregion search model with three processes modeled: examining the target of interest in narrow field of view, examining other target candidates in narrow field of view, and searching in wide field of view. This would lead to a three-exponent description analogous to the single-region search model.

Extensions of the model are possible if they are required by experimental data. One form of extension is to embed the model into a more global search strategy. For example, it has been suggested[1] that search could be modeled in three stages: (1) a preliminary orientation period; (2) a rapid search for highly visible targets; (3) a slower search for more difficult or hidden targets. The first stage can be represented by a simple delay (for example, exponentially distributed) which is convolved with the other stages (Appendix A). The second and third stages could be accommodated within the neoclassical

1 Witus (1993), for example.

model by adjusting the scale of the target transition rates (S and J). The second stage could be represented by choosing parameters to reduce the number of points of interest considered; the third stage would incorporate more points of interest.

Within a single Markov process model, additional states can be imagined. For example, the model described in the body of the paper does not explicitly include the physical processes of searching, that is, the time delays associated with moving the eye itself. This can be accommodated in the general framework by introducing new states; an approximate treatment is given in Appendix A. Other extensions include permitting nonexponential or nonstationary detection processes; as shown in Appendix D, these can be accommodated without changing the fundamental structure of the model.

The neoclassical model is simple enough to be falsifiable; that is, it has precise enough predictions that a comparison with experimental data will reveal its applicability and shortcomings. While a detailed validation requires specification of the S_i, J_i, U_i, and α_i rates (either from a vision model or direct measurement[2]), some partial validation of the assumptions or limiting cases is possible. There is support for the basic Markov structure of the overall search process in Harris (1993) who argues that a first order Markov process is the correct representation of free search (see Appendix A). In addition, it was assumed that the time spent examining a target candidate (as controlled by the rate W) was independent of the target characteristics. An ongoing IDA study of eye fixations [Rotman et al. (1993)] seems to confirm that the time spent on a target candidate is not strongly dependent on the target. The single target limit reduces to the previously introduced Smoke Model [Nicoll and Silk (1991)]; data analysis on smoke obscuration data may provide partial confirmation of the overall structure and is being pursued. Finally, some controlled clutter data is available in the HEL/IDA/NVEOD (Human Engineering Laboratory/ IDA/Night Vision and Electro-Optics Directorate) experiments and will provide eye fixation and timeline data [Birkmire et al. (1992)].

The neoclassical framework contains many of the insights gained from the long experience with the classical model approach and from the more recent work in modeling the human visual system. It is robust without being overly complicated and may prove to be a useful data analysis tool, as well as providing support to large-scale simulations such as JANUS and CASTFOREM.

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2 All of the search model parameters are transition rates which represent the average behavior of an ensemble of observers. There may be considerable variability in the underlying parameters that must be included in any complete description of human search performance.

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APPENDIX A

THE NEOCLASSICAL MODEL ASSUMPTIONS

APPENDIX A
THE NEOCLASSICAL MODEL ASSUMPTIONS

In this appendix, the underlying assumptions and parameterization of the neoclassical model are discussed in more detail.

A. MARKOV PROCESS ASSUMPTION

The fundamental premise of this paper is that the representation of the search process can be considered as a first order Markov process. This model was chosen in part for its relative simplicity and tractability and in order to relate it to the cueing probabilities as computed from an early vision model. However, there is independent evidence that this is not just a mathematical convenience but may be a reasonable representation of the search process. Harris (1993) analyzed some old (1935!) data on free-viewing scanning patterns. It was shown that time-reversed scanning patterns were statistically similar to the forward pattern, which is strong evidence for a first order Markov process. In addition, if a first order Markov process is assumed, this reversibility is a sufficient condition for stationarity of the search. Harris also showed empirically that the approach to the steady state was relatively rapid, supporting the separation in the neoclassical approach between fast and slow exponents. Harris concludes that "after a few saccades, the probability of fixating a feature reaches a steady value specific to that feature." Although the data are not recent and the application may differ, this is reasonable support for the approximations made. For a reversible system the transition rate matrix has the property that:

$$\frac{T_{ij}}{T_{ji}} = \frac{p_i^{eq}}{p_j^{eq}} \quad . \tag{A-1}$$

This condition is satisfied by the neoclassical approximations since the transition matrix only depends on the destination of the transition. However, the neoclassical matrix is not the most general satisfying Eq. (A-1). If a residual matrix is defined by

$$T_{ij} = p_i^{eq} R_{ij} \tag{A-2}$$

then the residual matrix for reversible search is symmetric:

$$R_{ij} = R_{ji} \quad . \tag{A-3}$$

The residual part of the transition rate is the same whether the observer is going from a weak to a strong target or from a strong to a weak. For the neoclassical model the residual matrix is chosen to be a constant; however, another choice would be to have it depend on the angular separation between the targets i and j, their relative eccentricity. Such a dependence surely exists and for a more elaborate Markov model would form an important part of the model specification. The psychophysical model of Witus (1993) endeavors to compute the eccentricity effects.

The Harris work suggests that the residual matrix may depend only on the relative eccentricity and not on the target signature characteristics. This implies that the approximation introduced by averaging over eccentricities is uncorrelated with the target and clutter signature properties. This provides some reassurance that no signature-related effects have been inadvertently overlooked.

B. FICTITIOUS vs. GENUINE QUIT STATES

This section discusses the use of fictitious absorbing or quit states and contrasts them with real quit states. The distinction can be important in the interpretation of data. The fictitious absorbing or quit state was introduced to evaluate the average over search paths used in Eq. (II-12), repeated here for convenience.

$$P_D(t) \quad = \left< 1 - e^{-\alpha_0 T(t)} \right> \quad . \tag{A-4}$$

The set of search paths over which the average is taken is independent of the target and α_0 since the search process is assumed to be independent of the detection process.

This sort of average is common in a variety of statistical problems. It can be computed by assigning a probability to each possible path through the Markov states and summing over all paths. For the continuous Markov process this leads to the so-called path integral since the number of possible paths is nondenumerably infinite. It is easier to follow the mathematics if a discrete Markov process is used with a finite time step:

$$\left< e^{-\alpha_0 \int_0^t ds\, \eta(s)} \right> = \sum_{\text{All paths}} \text{Prob(path)}\, e^{-\alpha_0 \int_0^t ds\, \eta_{\text{path}}(s)} \quad . \tag{A-5}$$

The probability of a particular path can be written down in terms of the transition rate matrix between states, T_{ij}. Divide the path into segments consisting of small time intervals Δt. Then a path can be identified by the states it occupies at successful moments. An arbitrary path is described by a sequence of states, beginning at time 0, $i_0 i_1 i_2 i_3 i_4$, and so forth. A path that starts in state 1, remains in it for 4 units of Δt, and then transitions to state 2 is described as 11112, and so on. The probability of a sequence of K steps is:

$$\text{Prob}(i_0 i_1 \ldots i_K) = \prod_{j=1}^{K} (\delta_{i_{j-1}, i_j} + \Delta t\, T_{i_{j-1}, i_j}) \quad . \tag{A-6}$$

This probability is correctly normalized. The transition matrix for a Markov process with no absorbing states has to conserve probability, namely

$$\sum_j T_{ij} = 0 \quad . \tag{A-7}$$

Summing over all paths of length K that begin with state i_0, one notes that the i_K index is free, that is, only appears in the last term of the product. Summing over this index gives a value of unity for the last term and frees up the index i_{K-1}. Repeating the argument, one sees that the sum over all paths of the probability given in Eq. (A-6) is unity.[1]

Returning to the average to be computed in Eq. (A-5), the integrand only is nonzero when the search is in the 0th state. In a sequence of states such as 1111222200000001, with N successive values of 0, the exponential contributes a factor of $e^{-N\alpha_0\Delta t}$. The weight of that path includes a factor of $(1 + T_{00}\,\Delta t)^{N-1} \approx e^{+(N-1)T_{00}\Delta t}$. As Δt becomes smaller, the value of N increases ($N\Delta t$ being the time spent in the 0th state) and the difference between N and N-1 can be neglected. The effect of the exponential factor on the sum is equivalent to replacing T_{00} by $T_{00} - \alpha_0$.

This changes the original probability conserving Markov process to one that loses probability from the 0th state. The average in (A-4) is replaced by

$$< e^{-\alpha_0 T(t)} >_{\text{Original Process}} = < 1 >_{\text{Modified Process}} \quad . \tag{A-8}$$

The average over "1" simply counts the fraction of paths that survive until time t, $\varepsilon(t)$.

$$<1>_{\text{Modified Process}} = \varepsilon(t) \quad . \tag{A-9}$$

The equations for the modified process are

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1 A continuum form of the path probability is given in Section III.

$$\dot{\varepsilon} = -\alpha_0 p_0 \qquad \text{(A-10a)}$$

$$\dot{w} = W\ p - S\ w + (W_0 - W)p_0 \qquad \text{(A-10b)}$$

$$\dot{p}_0 = S_0 w + J_0 p - J p_0 - W_0 p_0 - \alpha_0 p_0 \qquad \text{(A-10c)}$$

where, as discussed in Section II, a special value of W, W_0, is introduced for the P_0 state to allow for increased variability in the examination time of targets.

Eqs. (A-10) are identical to those for a system with a single real quit state. For some applications, a real quit state is an appropriate model of the search process. For example, consider the case of a single real target; if the clutter is such that it never generates false target detections, the observer may quit the search process when he finds the target. For multiple target cases, the observer is less likely to quit following a single detection.

The neoclassical model assumes that the detection process does not change the search in any way. Quitting the search process upon detection is only the most drastic method by which detection can affect search. In reality, there are a number of ways the sequence of detections may change the search process. All of these can be represented in the neoclassical framework by extensions of the model. The advantage of assuming that detection had no effect on search is that the equations for N target candidates always can be collapsed onto three equations; if complicated quit states are included in the model, the number of exponents will generally rise to N.

It is important to note that this separation between detection and search is an approximation that can have a real effect on the understanding and interpreting of data. For example, if the search process alone determines the time spent examining a target candidate (that is, if the absorbing state is fictitious), then the mean time for an examination is:

$$T_{\text{examination}} = \frac{1}{W_0 + J - J_0} \qquad \text{(A-11a)}$$

since the observer leaves the examination state at rate W_0 to the wandering state and at rate J-J_0 to other examining states. On the other hand, if the absorbing state is real, then the examination time is shorter since a visit may be truncated by a detection decision.

$$T_{\text{examination}} = \frac{1}{\alpha + W_0 + J - J_0} \ . \qquad \text{(A-11b)}$$

Since the examination time is directly measurable, there is a real difference in the determination of model parameters.

Using the time-sharing metaphor of Section II, the neoclassical model corresponds to a primitive autocratic time-sharing system. Each of the individual computations (users) is given a time allotment, depending on its priority but independent of the state of the calculation being done. As an extreme example of this, it does not even check to see if the calculation is complete: a certain time is allotted to it. In the search model this corresponds to the possibility of revisiting a target candidate even if a detection decision had been previously made. In a more elaborate time-sharing system the priorities of the different tasks are dynamically adjusted depending on the state of the tasks being done. For example, a task upon completion could signal the operating system that it was done and the master control algorithm would reallocate resources.

An intermediate case permits the reallocation of resources at the end of each allotted time interval. For this case, the average time of an examination is still given by Eq. (A-11a) but the system is "rebooted" with one less target after each detection. This nonstationary approach can be straightforwardly represented or simulated in the neoclassical framework. For example, the probabilities of detection of the various targets could be calculated as in the neoclassical model. At each time step, a random draw could be made to see if one or more of the targets are to be declared detected. If so, these targets are removed from the set of targets, and the equations solved with new initial conditions (including nonzero initial probabilities of detection for the other targets). Analysis of experimental data will be required to see if this extension is necessary.

C. NORMALIZING THE TRANSITION RATES

In principle, a complete human vision model would be able to predict all of the transition rates needed to specify the neoclassical model. More conservatively, we may expect that such models may accurately predict the relative attractiveness of different targets without providing an absolute normalization. For example, the human vision model may or may not consider all the possible cueing points in a completely symmetrical fashion—it is easier to describe the signature of a target (which may lead to a degree of attraction to the target and hence a transition rate) than it is to perform the same calculation for clutter. An isolated bush or tree possibly may be analyzed in an analogous fashion, but clumps of bushes, irregularities of the terrain, and so on raise fundamental difficulties in defining a cueing probability. If the human vision model considers each target or target-like object separately, the probabilities calculated may not include the competition between points of interest. One must be careful to not generate total rates S faster that $1/T_{sac}$. Typically, one would expect that only one decision to cease wandering and attend to a point of interest can

be made in T_{sac}; therefore, some overall normalization must be employed. Denote by $P^{raw}_{cue(i)}$ the unnormalized probability of cueing to the ith point of interest. If, consonant with the approximations made in the neoclassical model, the targets are considered to be independent and the cueing probabilities do not include competition directly, then the total probability of ceasing wandering and attending to some point of interest is

$$P = 1 - \Pi\,(1 - P^{raw}_{cue(i)}) \quad . \tag{A-12}$$

Then $P_{cue(j)}$ for the jth point of interest is

$$P_{cue(j)} = \frac{P^{raw}_{cue(j)}}{\sum_i P^{raw}_{cue(i)}} P \quad . \tag{A-13}$$

Alternately, one may put a constraint on S. The ratio $S/(S + W)$ is the fraction of time spent attending to target-like objects. If this is held fixed or used parametrically to define the search process and W is also specified, then:

$$S_j = \frac{P^{raw}_{cue}(j)}{\sum_i P(i)^{raw}_{cue}} S \quad . \tag{A-14}$$

D. USE OF W_0 vs. A CONSTANT W

The basic neoclassical model reduces to three equations only if a single value of W is used for all the targets. As noted in Section II, once the three equations are reached, one can artificially introduce a separate value for W that depends on the target of interest. This is generally an undesirable approach since it cannot be rigorously justified. However, the approximation of constant W must be evaluated against experimental data; if W varies widely then the use of a separate W for each target could be tried as an empirical approach. If one wishes to use Eq. (II-16) with a separate value of W_i for each point of interest, the value of W to be used for the remaining states must be an average or representative value. If we neglect jumping for simplicity, the following expression defines an effective W that preserves the right equilibrium values:

$$W = \frac{\sum_{j \neq 0} S_j}{\sum_{j \neq 0} \frac{S_j}{W_j}} \quad . \tag{A-15}$$

An uglier expression applies when jumping is not neglected:

$$\sum_{j\neq 0} \frac{S_i}{W_i + J} = \frac{\sum_{k\neq 0} S_k}{W + J} \; \frac{[1 + \frac{S_0 - J_0}{W_0 + J} - \sum_{k\neq 0} \frac{J_k}{W_k + J}]}{[1 + \frac{S_0 - J_0}{W_0 + J} - \sum_{k\neq 0} \frac{J_k}{W + J}]} \quad . \tag{A-16}$$

E. COGNITIVE vs. PHYSICAL STATES: DELAYS IN THE SEARCH PROCESS

The neoclassical framework describes the observer in terms of states that are primarily cognitive in nature; that is, they are defined more by the mental activity assumed to be occurring than by the distribution of fixations. For example, the state of attending to a point of interest requires that the observer be fixated on the target and attending to it as a target candidate. There are other states that need to be included if a complete physical and cognitive description is to be made. In particular, the model does not completely include the delays introduced by the physical time constants of the human visual system.

Consider, for example, an observer in the wandering state. The probability of staying in the wandering state is:

$$P_W(t) = e^{-S t} \quad . \tag{A-17}$$

This expression describes a smooth decay out of the wandering into one of the examining states. The probability that the transition out of w will occur in an interval dt around t is:

$$p_{w->}(t)dt = S\, e^{-S t} dt \quad . \tag{A-18}$$

Note that although the average time until a transition is $1/S$, transitions after an arbitrarily short time are allowed and the most probable time of transition is $t = 0$! This contradicts common sense and experience since no change in the physical state of the eye can happen in less than approximately 0.1 second.

The problem arises because the process of physically moving the eye has no explicit representation. This can be incorporated into the Markov framework by adding delay states between the cognitive states. Consider a simple example of one state, w, making transitions into another state, s, via an intermediate state, d:

$$\dot{w} = -S\,w$$

$$\dot{d} = +S\,w - \eta d \qquad \text{(A-19)}$$

$$\dot{p} = \eta d \quad .$$

If η is very large, then the intermediate state drains rapidly, $d \approx 0$, and $S\,w \approx \eta d$. The equations are effectively:

$$\dot{w} = -S\,w$$

$$\dot{p} = S\,w \quad . \qquad \text{(A-20)}$$

However, in general, the solution to Eq. (A-20) will be:

$$p(t) = 1 - \frac{\eta}{\eta - S}\, e^{-S\,t} + \frac{S}{\eta - S}\, e^{-\eta t} \quad , \qquad \text{(A-21)}$$

and while the transition time distribution out of the w state is still given by Eq. (A-19) the transition rate into p is:

$$p_{->s}(t)dt = \frac{\eta S}{\eta - S}\,[e^{-S\,t} - e^{-\eta t}]dt \quad . \qquad \text{(A-22)}$$

These are illustrated in Fig. A-1 for $S = 2$ and $\eta = 10$. The transition out of w shows no delay while the transition into p is delayed about 2/10 of a second.

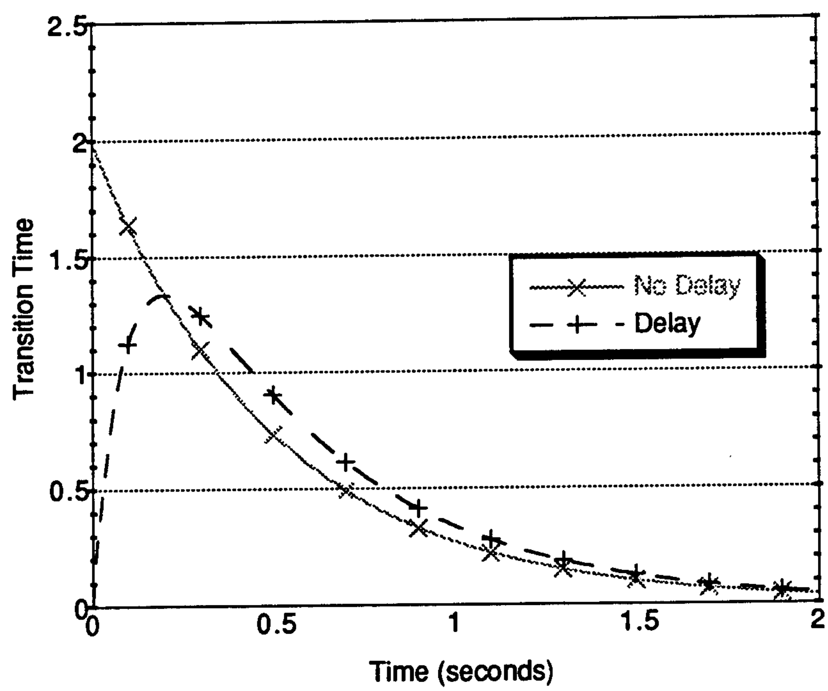

Figure A-1. Transition Times With and Without Delay

The curve exhibiting the delay is the more appropriate and physical curve. However, changing the neoclassical equations to incorporate these delays in general form would be unduly complicated and unnecessary except for the analysis of very short times. An approximate treatment is therefore more appropriate.

If physical delay states were introduced between all the transitions between cognitive states, the number of Markov states would increase drastically. In principle, for N targets there would be N delay states between wandering and the POI, N delay states between the POI and wandering, and N(N-1) delays between the POI! Even if delays were inserted into the three effective state representation, there would be seven new states (Fig. A-2). There would be ten exponents and initial conditions to fix. However, if we imagine that the delays are all short then the form of the results is clear even without writing out the equations.

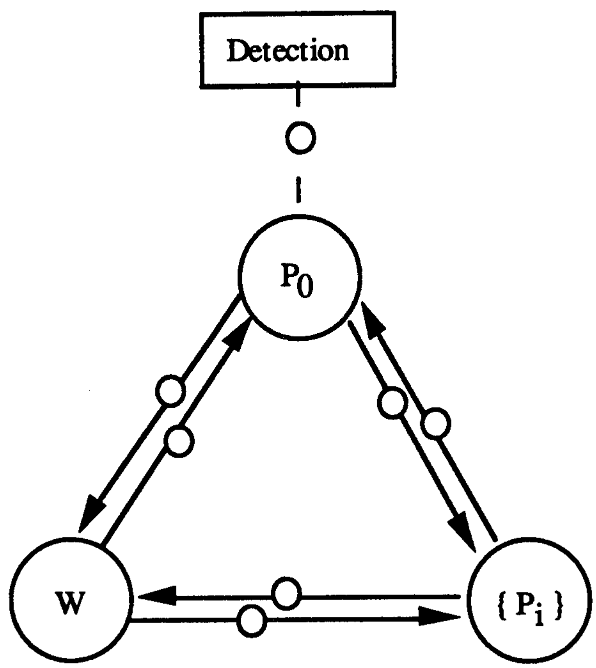

Figure A-2. Adding Delay States to the Neoclassical Model

There will be 7 very fast exponents, corresponding to the draining of the delay states. These will all be of the same order, $\eta \approx 10$; as a practical matter these can be lumped into a single exponent. The value of the remaining three exponents will only differ slightly from the values in the model without the delay states. Thus, the probability of detection (in the single exponent approximation) will be of the form:

$$P_D(t) \approx 1 - a\,e^{-\lambda_3 t} - (1-a)e^{-\eta t} \quad . \tag{A-23}$$

The value of the constant can be fixed by imposing the initial condition that there is a delay before the first detection—the probability of detection curve is of zero slope at the origin.

$$P_D(t) \;=\; 1 - \frac{\eta}{\eta - \lambda_3} e^{-\lambda_3 t} + \frac{\lambda_3}{\eta - \lambda_3} e^{-\eta t} \quad . \tag{A-24}$$

Note that this is of the same form as the single delay state described in Eq. (A-21). This form is also equivalent to convolving the probability of detection with an exponentially distributed delay with mean value of $1/\eta$.

If all three exponents are used (for example, in describing short times), it becomes harder to guess the initial conditions. One way of modeling the effect is to imagine that the delays occur in the space of eigenstates that have simple exponential behavior. The probability of detection derived in the neoclassical framework is mathematically equivalent to three states, which empty into the detection state; if we add a delay state to each mode, inescapable delays can be represented (Fig. A-3).

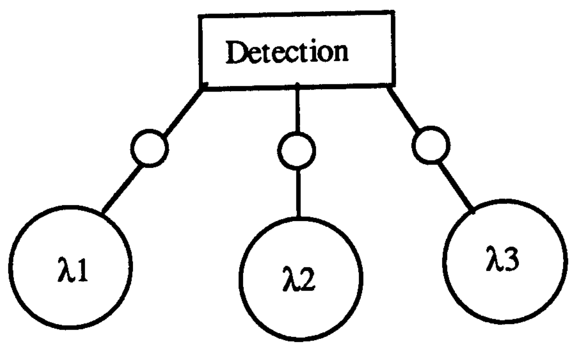

Figure A-3. Adding Delays to the Eigenmodes

The probability of detection is then given by convolving delays on each of the eigenstates:

$$P_D(t) = \sum_{i=1}^{3} e_i \left(1 - \frac{\eta_i}{\eta_i - \lambda_i} e^{-\lambda_i t} + \frac{\lambda_i}{\eta_i - \lambda_i} e^{-\eta_i t}\right) \quad . \tag{A-25}$$

In Eq. (A-25) we have allowed for the possibility of different values of η_i in each of the channels; for simplicity a single value of $\eta \approx 10$ seconds^{-1} may suffice. The introduction of the physical delays does not change the results given in the body of the paper except for short times. For example, the mean time to detect is now given by:

$$< t_D > = \sum_{i=1}^{3} e_i \left(\frac{1}{\lambda_i} + \frac{1}{\eta_i}\right) \quad . \tag{A-26}$$

If a single value of η is used, the mean time to detect is increased by $1/\eta$.

The same approach applies to any quantity of interest; the appropriate neoclassical model is used to calculate the results in the absence of delay; the result is then convolved with an exponentially distributed delay to account for any latency in the process. Thus, the distribution of times spent in the wandering state is to be taken to be Eq. (A-21). This method of representing physical delays introduces one or more parameters to describe the delays, but these should be independent of scenario since the delays are physical in nature.

In addition to these delays that occur throughout the search process, there can be start-up or orientation delays that apply only to the beginning of search. These can be represented in the same way by convolving the results with the probability distribution of the start-up delay.

F. HOW MANY STATES—TWO EXPONENTS VERSUS THREE

The Markov framework of the neoclassical model allows for an arbitrary number of exponents equal to the number of states that are included in the state description. The choices made for the neoclassical equations described in the main text were based on two relatively straightforward approximations.

1. The observer is not always examining potential targets. Thus, there is a fundamental distinction between examininination states and nonexamination states. The approximation made in the neoclassical model is to replace all the nonexamination states with a single, "wandering" state.
2. The transition rates into a particular state generally depend on the current state of the process. The approximation made in the neoclassical model is to retain only two distinct rates into an examination state: S_0, the transition from wandering, and J_0, the transition rate from any other examining state. A second approximation is to use only a single rate, W, from examining states to wandering.

These approximations lead to the three-exponent model. A crucial element of the model is the distinction between the $S_0$and J_0 transition rates. The cognitive state of the observer is assumed to be different during wandering and examining phases of the search, which implies that these two rates should be unequal with the more likely result being $J_0 < S_0$. However, a large simplification in the model is made if it is instead assumed that these rates are instead equal: $J_0 = S_0, J = S$. In this case, the distinction between wandering and examining states, although real, is effectively eliminated.

Examining the neoclassical equations for equal rates: $\lambda_1 = R$, and $e_1 = 0$, exactly. The remaining eigenvalues and amplitudes are:

$$\lambda_{2,3} = \frac{W + S + \alpha_0 \pm \sqrt{(W + S + \alpha_0)^2 - 4\alpha_0 S_0}}{2} \tag{A-27a}$$

$$e_i = \frac{\lambda_j - \alpha_0 p_0(0)}{\lambda_j - \lambda_i} \quad . \tag{A-27b}$$

This two-exponent model closely resembles the classical model. For example, the time of first arrival to a point of interest has the probability distribution:

$$P_a(t) = p_0(0) + (1 - p_0(0))e^{-S_0 t} \quad , \tag{A-28}$$

which is identical to Eq. (I-7). The distinction arises since this model allows for the possibility that the observer will not detect the object immediately upon first arrival.

The difference between the two-exponent and three-exponent models is primarily in the distribution of first arrival times. The probability of detection curves after first arrival time (obtained by setting $p_0(0) = 1$ in the general expressions) are very similar. For the three-exponent model, the amplitudes are given by:

$$e_i = \frac{(\lambda_i - R)(\lambda_i - (W + J))}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} \quad . \tag{A-29}$$

For small values of λ_3, e_1 is small and for both the two- and three-exponent cases, the probability of detection after first arrival is well approximated by a two-exponent expression:

$$P(t) = 1 - P_{visit} e^{-(W+J+\alpha_0)t} - (1 - P_{visit})e^{-\lambda_3 t} \quad . \tag{A-30}$$

Given the limitations of any practical validation experiment, only the time of first arrival will provide a method of distinguishing between the two models within the neoclassical framework.

APPENDIX B

FURTHER EXAMPLES OF THE NEOCLASSICAL MODEL

APPENDIX B
FURTHER EXAMPLES OF THE NEOCLASSICAL MODEL

This appendix includes examples of special cases of the neoclassical model.

A. WANDERING START

One of the most natural initial conditions that could be adopted for general modeling is to assume that the observer is not attending to any of the target or clutter points at t = 0. This corresponds to w(0) = 1, and p(0) = $p_0(0)$ = 0. For $W = W_0$, to leading order in J_0 and S_0, and using Eq. (II-6) (namely $J_0 = \kappa S_0$, $J = \kappa S$), the equations give:

$$P_D(t) = [1 + \frac{\alpha_0 S_0[(W+J)^2 - \alpha_0(S-J)]}{(W+S)^2(W+J+\alpha_0)^2}](1 - e^{-\lambda_3 t})$$

$$+ \frac{\alpha_0 S_0}{S - J - \alpha_0}[\frac{\alpha_0}{(W+J+\alpha_0)^2}(1 - e^{-(W+J+\alpha_0)t})$$

$$- \frac{S-J}{(W+S)^2}(1 - e^{-(W+S)t})] \quad . \qquad \text{(B-1a)}$$

The amplitude of the slow (λ_3) exponent term is greater than one if $(W+J)^2 > \alpha_0 (S-J)$; the λ_2 and λ_1 terms always have different signs. The latter terms compete: the details of the search parameters determine the relative sizes.

B. QUASI-EQUILIBRIUM

Another mathematically reasonable set of initial conditions is to assume that the observer begins in the equilibrium state. That is, there is a nonzero probability that the observer is fixed on the ith point of interest given by (for $W = W_0$)

$$p_i(0) = S_i / R; \qquad w(0) = W/R \quad . \qquad \text{(B-2a)}$$

In this case the amplitudes are:

$$e_i = \frac{\alpha_0[S_0 W + J_0 S][\lambda_i - W - J][\lambda_i - R]}{(W + J)R\, \lambda_i (\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} \quad . \tag{B-2b}$$

If we adopt $J_0 = \kappa S_0$, $J = \kappa S$, a slightly simpler form is obtained:

$$e_i = \frac{\alpha_0 S_0 [\lambda_i - W - J][\lambda_i - R]}{R\, \lambda_i (\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} \quad . \tag{B-2c}$$

The e_1 term is $O(S_0^2)$ since the unperturbed value of λ_1 is R, and that the e_2 term is $O(\alpha_0^2)$ since the unperturbed value of λ_2 is $W + J$. For low-observable, hard-to-cue targets, S_0 is small. Working to lowest order in S_0, the value of e_1 is negligible using these equilibrium values for the initial conditions. To the same order of accuracy, one can set $e_1 = 0$ identically. This gives a linear relationship between $w(0)$ and $p_0(0)$, which is only that of equilibrium to leading order in S_0; a quasi-equilibrium approximation is defined by adopting $e_1 = 0$ without regard to the order of S_0. In this case, there are only two exponents and amplitudes remaining. The exponents must be determined from the full cubic equation, but the expression for the amplitudes e_2 and e_3 is simplified:

$$e_i = \frac{\lambda_j - \alpha_0\, p_0(0)}{\lambda_j - \lambda_i} \quad . \tag{B-2b}$$

C. THE STATE WITH NO RETURN

If $W_0 = J = J_0 = 0$, the state, once entered, does not allow any return to other states. The general equations again simplify. The second exponent $\lambda_2 = \alpha_0$ exactly and the remaining exponents are a quadratic with roots given by:

$$\lambda_{1,3} = \frac{R \pm \sqrt{R^2 - 4S_0 W}}{2} \quad . \tag{B-3a}$$

This is the limit of the arrival time distribution for $J_0 = 0$, so that λ_3 is approximately equal to the rate of first arrival. For small S_0, these reduce to

$$\lambda_1 \approx R \; ; \; \lambda_3 \approx S_0 W / R \quad . \tag{B-3b}$$

Neglecting the λ_1 term for simplicity, the probability of detection is

$$P_D(t) = p_0(0)\,(1 - e^{-\alpha_0 t})$$
$$+ \frac{1 - p_0(0)}{\alpha_0 - \lambda_3}\,[\alpha_0(1 - e^{-\lambda_3 t}) - \lambda_3\,(1 - e^{-\alpha_0 t})\,] \quad . \qquad \text{(B-3c)}$$

If α_0 gets large, this reduces to the detect-when-cued example. The same result can be obtained by integrating over the probability of first arrival times and then using the exponential detection model with rate given by α_0.

D. NO WANDERING

This case considers the limit of $W_0 = W = 0$; the observer never leaves a point of interest to wander and spends all of his time examining target candidates. For this case, $\lambda_1 = S$ exactly; the remaining exponents are a quadratic whose roots are

$$\lambda_{2,3} = \frac{J + \alpha_0 \pm \sqrt{(J + \alpha_0)^2 - 4J_0\alpha_0}}{2} \quad . \qquad \text{(B-4a)}$$

For small J_0, this reduces to

$$\lambda_2 \approx J + \alpha_0\,;\ \lambda_3 \approx \frac{\alpha_0 J_0}{J + \alpha_0} \quad . \qquad \text{(B-4b)}$$

For large α_0, Eq. (B-4a) becomes

$$\lambda_2 \approx \alpha_0\,;\ \lambda_3 \approx J_0 \quad . \qquad \text{(B-4c)}$$

The amplitudes are given in general by

$$e_1 = \frac{\alpha_0(J_0 - S_0)\,w(0)}{(S - \lambda_2)(S - \lambda_3)} \qquad \text{(B-4d)}$$

and for e_i, $i = 2,3$,

$$e_i = \frac{S\lambda_j - \alpha_0[J_0\,p(0) + S_0\,w(0)] + \alpha_0 p_0(0)(\lambda_i - S)}{(\lambda_i - S)(\lambda_i - \lambda_j)} \quad . \qquad \text{(B-4e)}$$

If the observer cannot even start in the wandering state, $w(0) = e_1 = 0$ and this simplifies to

$$e_i = \frac{\lambda_j - \alpha_0\,p_0(0)}{\lambda_j - \lambda_i} \quad . \qquad \text{(B-4f)}$$

E. SHORT TIMES IN LONG SEARCHES

The examples discussed in the main body of the paper show that for long field-of-view searches the long time behavior is controlled by λ_3 and the initial conditions (barring an abnormally high probability of being initially cued to the target) are not of great importance. This is not the case for short times, namely for the first few examinations of points of interest. All three exponentials are required to fully describe the short time behavior. Expanding the general result for small values of t:

$$P_D(t) = p_0(0)\,\alpha_0\, t + \alpha_0[J_0\, p(0) + S_0\, w(0) - \hat{W}_0\, p_0(0)]\frac{t^2}{2} + \ldots \quad . \qquad \text{(B-5a)}$$

Figure B-1 shows the results for $J_0 = J = 0, W_0 = W = 1, S = 1, S_0 = 1/10$ and $\alpha_0 = 2$) and a number of different initial conditions [the equilibrium conditions are w(0) = 0.5, p(0) = 0.5 and $p_0(0) = 0.05$]. For wandering state initial conditions, the curve begins below the single exponent approximation and then crosses over at about 1 second. All of the curves become parallel to the single-exponent approximation; representing a simple time shift (compare to Table III-1). This time shift can be an important fraction of the total in the short time regime. For example, the time to reach a probability of detection of 0.1 varies from 2.5 to 4 seconds.

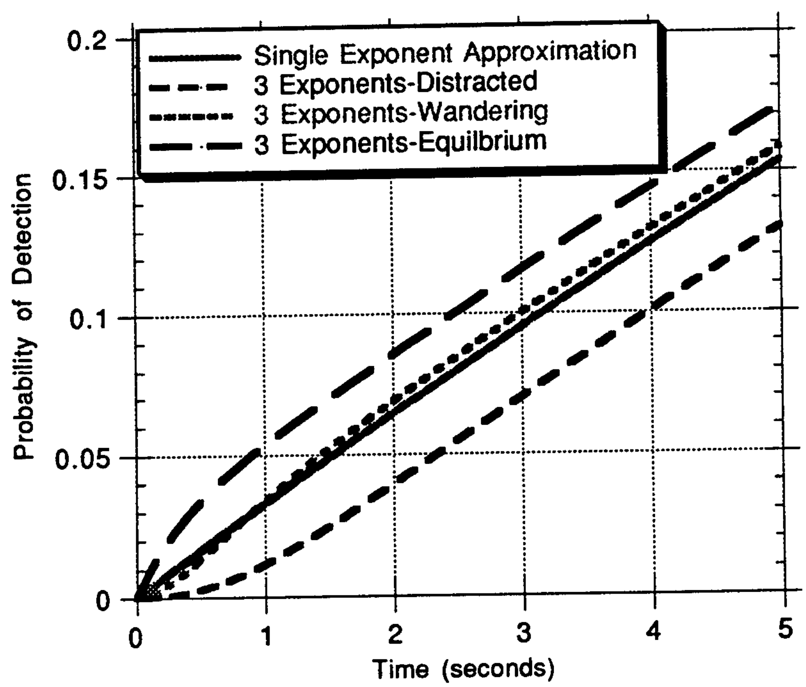

Figure B-1. Short Time P_D Behavior for Different Initial Conditions

If the observer has some chance of attending to the target at $t = 0$, $[p_0(0) \neq 0]$ then the probability increases linearly with time. On the other hand, if $p_0(0) = 0$ (that is, if there is no probability of attending to target at $t = 0$), $P_D(t)$ is quadratic in time with a curvature determined by the cueing rate. There is an initial delay before detection seems to get going. If the observer is attending to a distracter at $t = 0$ (either a clutter object or another target), then $p_0(0) = w(0) = 0$ and $P_D(t)$ is again quadratic but with a smaller coefficient since the observer must jump from the distracting point of interest to the target. If jumping between points of interest is precluded, then the initial form of the probability of detection will be cubic! These delays are commonly observed in search and detection experimental data [see Blecha et al. (1991), Nicoll (1992)]. However, as discussed in Appendix A, there are a number of different mechanisms that can be responsible for a delay all of which may need to be considered in greater detail.

One way of re-expressing the dependence of the short time behavior of the neoclassical model is to consider the hazard function. The hazard function is the rate of detection at time t given that no detection has occurred earlier. Formally it is:

$$H(t) = \frac{\dot{P}_D(t)}{1 - P_D(t)} = \frac{\sum_i e_i \lambda_i e^{-\lambda_i t}}{\sum_i e_i e^{-\lambda_i t}} . \qquad \text{(B-5b)}$$

For a single exponential, $H(t) = 1/\tau$, where τ is the time constant. For large values of t, this will be dominated by the slowest exponent, $H(t) \approx 1/\lambda_3$. For small times:

$$H(t) = \frac{p_0(0)\, \alpha_0 + \alpha_0 [J_0\, p(0) + S_0\, w(0) - \hat{W}_0\, p_0(0)]\, t + \ldots}{1 - p_0(0)\, \alpha_0\, t + \ldots} . \qquad \text{(B-5c)}$$

The hazard functions for the cases illustrated in Fig. B-1 are shown in Fig. B-2. For the values chosen, the hazard reaches the long time value after about two seconds, but is highly dependent on the initial conditions at earlier times.

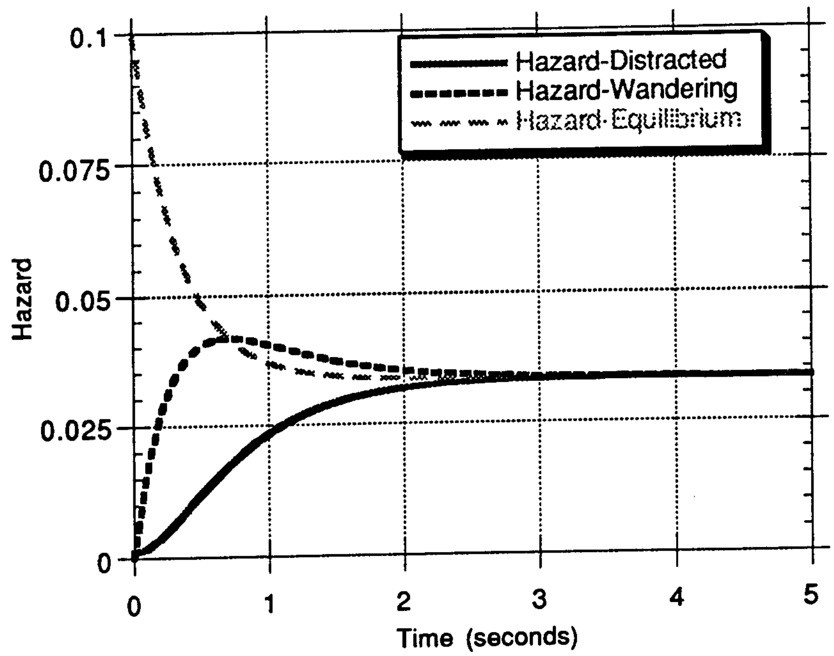

Figure B-2. Hazard Functions

F. QUITTING FROM THE WANDERING STATE

In this example of quitting the search process, it is assumed that the observer only quits from the wandering state and not from the various points of interest. The relevant equations in that case are

$$\begin{aligned} \dot{\varepsilon} &= -\alpha_0 p_0 - Q\,w \\ \dot{w} &= W\,p - S\,w - Qw - (W_0 - W\,)p_0 \\ \dot{p}_0 &= S_0\,w + J_0\,p - J\,p_0 - W_0\,p_0 - \alpha_0 p_0 \\ \dot{q} &= w\,Q\ , \end{aligned} \qquad \text{(B-6a)}$$

where q is the probability that the observer has quit. The quit state variables must be included explicitly; the probability of detection being given by $P_D(t) = 1 - \varepsilon(t) - q(t)$. Fortunately, a quadratic equation for the eigenvalues can be avoided because of remaining symmetries of the equations allow the q terms can be separated algebraically. The eigenvalues are determined from the cubic equation.

$$\lambda\,[(\lambda - (R + Q))(\lambda - \hat{W}_0) - \Delta_0 \delta W + \alpha_0 J_0] + Q\,[W\,(\lambda - \hat{W}_0) - \delta W J_0] - \Pi - Q\alpha_0 J_0 = 0\ . \qquad \text{(B-6b)}$$

The probability of detection is

$$P_D(t) = \sum_{i=1}^{3} d_i (1 - e^{-\lambda_i t}) \quad , \tag{B-6c}$$

where (after some algebra) the coefficients are

$$d_i = e_i \frac{\Pi - \alpha_0 J_0 (\lambda_i - Q)}{\Pi_Q - \lambda_i (WQ + \alpha_0 J_0)} \tag{B-6d}$$

$$e_i = \frac{\frac{\Pi_Q}{\lambda_i} - QW - w(0)[\alpha_0 S_0 + Q(\hat{W}_0 - \lambda_i)] - p(0)\alpha_0 J_0 + p_0(0)[\alpha_0 (\lambda_i - R - Q) - Q\delta W]}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} \tag{B-6e}$$

where the product of the eigenvalues is now

$$\Pi_Q = \Pi + Q\,[W\hat{W}_0 + J_0 \delta W] + QJ_0\alpha_0 \quad . \tag{B-6f}$$

For this version of quitting, P_∞ is:

$$P_\infty = \frac{\Pi + \alpha_0 Q\,[J_0\, p(0) + W\, p_0(0)]}{\Pi_Q} \quad . \tag{B-6g}$$

The difference between simple quitting and quitting from the wandering state can be illustrated by considering the limit of large Q—fast quitting. For simple quitting, all the exponents are approximately equal to Q and $P_D(t) \approx P_\infty (1 - e^{-Q\,t})$ where:

$$P_\infty \approx \sum_{i=1}^{3} e_i \left[\frac{\lambda_i}{Q} - \left(\frac{\lambda_i}{Q}\right)^2\right]$$

$$= \frac{\alpha_0\, p_0(0)}{Q} + \frac{\alpha_0\,[J_0\, p(0) + S_0\, w(0) - (W_0 + J + \alpha_0) p_0(0)]}{Q^2} \quad . \tag{B-6h}$$

As $Q \Rightarrow \infty$, $P_\infty \Rightarrow 0$ rapidly. For quitting from the wandering state, the observer does not quit from the point of interest states. As long as the observer stays within the point of interest states, the search continues. Only one of the exponents becomes large: $\lambda_1 \Rightarrow \infty$. The other exponents satisfy a quadratic and are

$$\lambda_{2,3} = \frac{W + W_0 + J + \alpha_0 \pm \sqrt{(\delta W + \alpha_0)^2 - 4J_0(\delta W + \alpha_0)}}{2} \quad . \tag{B-6i}$$

For small values of J_0 these exponents become

$$\lambda_2 = W + J_0 ; \; \lambda_3 = W_0 + J - J_0 + \alpha_0 \, , \tag{B-6j}$$

which are the transition rates out of the distracter and target states, respectively. The amplitude coefficients and P_∞ are:

$$d_i = p(0) \frac{\alpha_0 J_0}{\lambda_i(\lambda_j - \lambda_i)} + p_0(0) \frac{\alpha_0 (W - \lambda_i)}{\lambda_i(\lambda_j - \lambda_i)} \tag{B-6k}$$

$$P_\infty = \alpha_0 \frac{J_0 \, p(0) + W \, p_0(0)}{W (W_0 + J + \alpha_0) + J_0(W_0 - W + \alpha_0)} \, .$$

APPENDIX C

MULTITARGET CORRELATIONS

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MULTITARGET CORRELATIONS

As noted in the body of the paper, the probabilities of detection for different targets are not independent, but are correlated by the search process. This dependence of detection on the search should not be confused with a dependence of the search on the detection of one target, which subsequently leads to an effect on detection of another target. The latter effect, which is important for the consideration of target arrays, is not directly treated within the neoclassical framework (target arrays may be partly included by constructing "super-targets," which represent the array as a whole as well as having points of interest representing the single targets).

The search process indirectly introduces correlation between the detections of targets simply because the observer cannot be in two places at once. The detection process is modeled by considering the time spent on target. For example, the probability that neither of two targets has been detected is:

$$P_2(t, \alpha_0, \alpha_1) = < e^{-\alpha_0 T_0(t)} e^{-\alpha_1 T_1(t)} > \qquad \text{(C-1)}$$

where $T_0(t)$ and $T_1(t)$ are the time spent on targets 0 and 1, respectively, and α_0 and α_1 are the detection rates for the targets. The average is taken over all possible search paths. When a search visits target 0, $T_0(t)$ increases; when the search visits target 1, $T_1(t)$ increases. The two processes are inextricably intertwined. However, the same approach that was used in Section II to evaluate the probability of detection for a single target can be used to evaluate expressions such as Eq. (C-1).

All of the correlation effects can be derived from the multiobject generating function. Consider m targets (numbered from 0 to m-1). The probability that the observer has declared none of the m objects to be a target at time t, $P_m(t)$, is the average over all search paths of a simple product of exponentials:

$$P_m(t, \alpha_0, \alpha_1, \ldots \alpha_{m-1}) \; = < \; e^{-\int_0^t \alpha_0 \eta_0(s) ds} \, e^{-\int_0^t \alpha_1 \eta_1(s) ds} \ldots e^{-\int_0^t \alpha_{m-1} \eta_{m-1}(s) ds} > \qquad \text{(C-2)}$$

where the random function η_i controls the detection for the ith target: $\eta_i(t) = 1$ whenever the search visits the ith target candidate and $\eta_i(t) = 0$ otherwise. Knowledge of the P_m permits the computation of any multitarget probability. For example, the probability that either target 0, target 1, or target 2 has been detected is

$$P_D^{0 \text{ or } 1 \text{ or } 2}(t) = 1 - P_3(t, \alpha_0, \alpha_1, \alpha_2) \ . \tag{C-3}$$

Recall that the neoclassical model is fundamentally a single-exponent model in terms of the time on target; the three exponents arise from taking the average over search paths. Similarly, in this case, it is the average over search paths that introduces the complexity for the multitarget correlation function.

If the detections were statistically independent then Eq. (C-2) would factor:

$$\begin{aligned} &< e^{-\int_0^t \alpha_0 \eta_0(s) ds} \, e^{-\int_0^t \alpha_1 \eta_1(s) ds} \cdots e^{-\int_0^t \alpha_{m-1} \eta_{m-1}(s) ds} > = \\ &\quad < e^{-\int_0^t \alpha_0 \eta_0(s) ds} >< e^{-\int_0^t \alpha_1 \eta_1(s) ds} > \cdots < e^{-\int_0^t \alpha_{m-1} \eta_{m-1}(s) ds} > \ . \end{aligned} \tag{C-4}$$

As will be shown below, in the single-exponent approximation to the individual detections, this factorization does hold approximately for small values of m, so that the individual target detections are nearly statistically independent. However, for large m the approximation breaks down and the correlations have to be included.

Equation (C-2) is solved exactly as in the single target case: m absorbing states are introduced and a system of m + 2 linear differential equations is solved to determine the m + 2 exponents of the system. The exponents are the roots of the following equation of degree m +2 :

$$\lambda(\lambda - R) - \sum_{i=0}^{m-1} \frac{\alpha_i [W S_i + J_i (S - \lambda)]}{\lambda - \hat{W}_i} = 0 \tag{C-5a}$$

where

$$\hat{W}_i = W + J + \alpha_i \ . \tag{C-5b}$$

The correlation function $P_m(t)$ is a linear combination of the m + 2 exponents depending on the initial conditions just as in the m = 1 case.

There is a special case of these equations that can be used to clarify the computation of the m-target generating functions. If the detection rates for the m targets being considered are all the same, $\alpha_i = \alpha$, then the equations collapse to the three-exponent neoclassical model with effective values of the transition rates into the target of interest:

$$J_0^{eff(m)} = \sum_{i=0}^{m-1} J_i \; ; \; S_0^{eff(m)} = \sum_{i=0}^{m-1} S_i \quad . \tag{C-6}$$

The entire formalism developed in the body of the paper can be applied using these effective rates. For small values of m, the effective transition rates will still be small and the perturbation results of the m = 1 case can be applied. The m-target correlation function will be dominated by a single slow exponent, which will be proportional to the effective rates and will therefore just be the sum of the individual single target detection rates. In this case, the factorization does work and the probabilities of detection are independent, as illustrated in Eq. (C-7):

$$\begin{aligned} &< e^{-\int_0^t \alpha\eta_0(s)ds} \, e^{-\int_0^t \alpha\eta_1(s)ds} \, ... \, e^{-\int_0^t \alpha\eta_{m-1}(s)ds} > \approx e^{-\lambda_3^{eff} t} = e^{-\sum_{i=0}^{m-1} \lambda_3^{(target\ i)} t} \\ &\approx < e^{-\int_0^t \alpha\eta_0(s)ds} >< e^{-\int_0^t \alpha\eta_1(s)ds} > .... < e^{-\int_0^t \alpha\eta_{m-1}(s)ds} > \quad . \end{aligned} \tag{C-7}$$

However, as m grows larger the effective transition rates are no longer small: for m = N = number of targets, they are equal to J and S. In this limit, the independence approximation generally will break down. This is illustrated in Figs. C-1 and C-2 for $W = 1$, $S = 1$, $J = 0$, and 10 targets each with $S_i = 1/10$. The figure compares $P_m(t)$ with $P_1(t)^m$. These would be equal in this case if statistical independence held exactly. For simplicity, wandering initial conditions are assumed for all cases shown.

The correlation functions are plotted with a reversed y-axis to make them resemble the detection probability curves. For m = 2 and m = 4, there is very little difference in the exact result and the result assuming independence. However, for m = 6 and m = 8 there are considerable discrepancies as shown in Fig. C-2. Finally, Fig. C-3 compares the prediction for m = 10. Note that since there are ten targets, this case gives the probability of detection of the first target detected. In this case the discrepancies are larger but not overwhelming. Note that the time scales of the figures differ; as m increases, the time scale for the probability of detecting one of the m targets decreases.

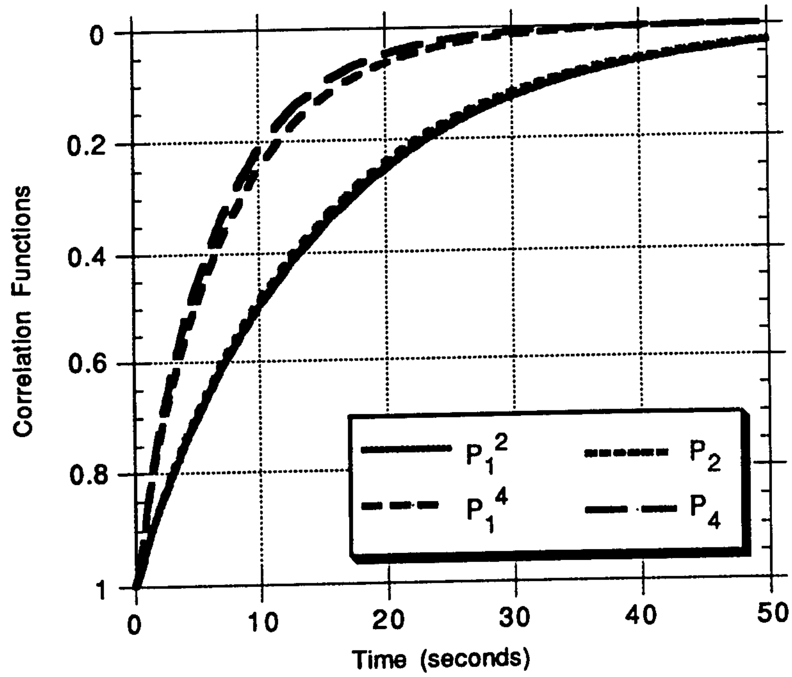

Figure C-1. m-Target Function Comparison: m = 2 and m = 4

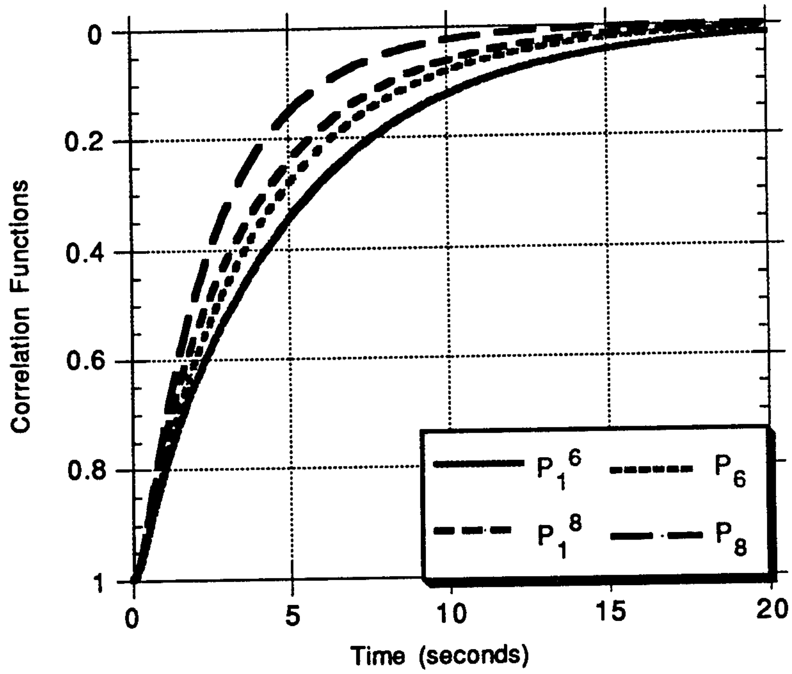

Figure C-2. m-Target Function Comparison: m = 6 and m = 8

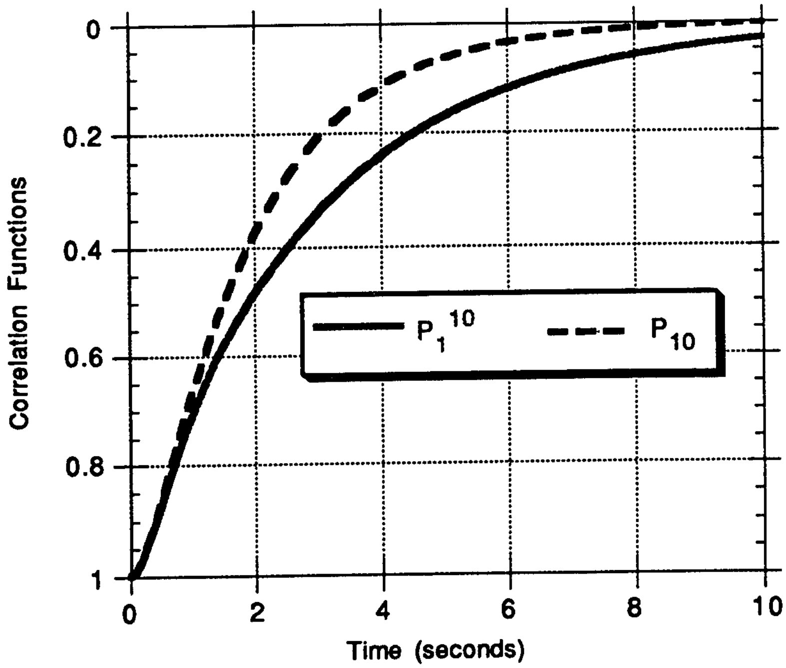

Figure C-3. m-Target Function Comparison: m = 10

These results for the equal α case give the correct qualitative description of the correlations for the general case. For single-target detection probabilities that are dominated by a slow exponent, the multitarget probabilities will appear to be independent as long as the sum of the slow exponents for each target is not large. When this breaks down, the full system of equations must be considered more carefully.

Returning to the general case, P_m can be evaluated perturbatively for small detection rates ($\alpha_i << W$). There will be m + 1 fast eigenvalues and 1 slow eigenvalue. The fast eigenvalues are

$$\lambda_R = R + \sum_{i=0}^{m-1} \frac{\alpha_i W (S_i - J_i)}{R(R - \hat{W}_i)} \quad . \tag{C-8a}$$

This corresponds to λ_1 for the m = 1 case. There are m exponents similar to λ_2.

$$\lambda^{(i)} = \hat{W}_i - \frac{\alpha_i [W S_i + J_i (S - \hat{W}_i)]}{\hat{W}_i (R - \hat{W}_i)} \quad . \tag{C-8b}$$

Finally, the slow exponent is just the sum of the individual slow exponents for the m targets:

$$\lambda_{slow} = \sum_{i=0}^{m-1} \frac{\alpha_i [W S_i + J_i S_i]}{R \hat{W}_i} . \qquad \text{(C-8c)}$$

For long times, the m-target function is dominated by the smallest exponent just as in the single target case. In that limit, $P_m(t)$ will be proportional to the product of the individual single target factors that will, *a fortiori*, be dominated by their slow exponents. For large m the value of the λ_{slow} may no longer be given by the perturbative approximation (even if the individual single-target exponents are themselves small), and the behavior even at long times will be affected.

For short times, the initial conditions of the search may become important. In general, for very short times, there is effective independence shown by the m-target generating function (essentially since the search has had time to produce correlations); the Taylor series of the difference is

$$P_n(t,\alpha_0,\alpha_1,\ldots\alpha_{m-1}) - \prod_{i=0}^{m-1} P_1(t,\alpha_i) = - \sum_{i \neq j} \alpha_i \alpha_j p_i(0) p_j(0)\, t^2/2 + O(t^3) , \qquad \text{(C-9)}$$

so that in the wandering state used in the example given above, the agreement is up to cubic terms. The effect of the initial conditions can be explored in more detail by computing P_m explicitly. In the perturbative realm and assuming that the initial probability of fixating on one of the targets is not abnormally large, the coefficients are

$$e_R = \sum_{i=0}^{m-1} \frac{\alpha_i [w(0) S_i + p(0) J_i] - \hat{W}_i \lambda_{slow}^{(i)}}{R (\hat{W}_i - R)} \qquad \text{(C-10a)}$$

$$e_{(i)} = \frac{R \lambda_{slow}^{(i)} - [w(0) S_i + p(0) J_i] + \alpha_i (\hat{W}_i - R)\, p_i(0)}{\hat{W}_i (\hat{W}_i - R)} \qquad \text{(C-10b)}$$

$$e_{slow} = 1 - e_R - \sum_{i=0}^{m-1} e_{(i)} , \qquad \text{(C-10c)}$$

where the individual slow exponents are

$$\lambda_{slow}^{(i)} = \frac{\alpha_i [W S_i + J_i S_i]}{R \hat{W}_i} . \qquad \text{(C-10d)}$$

In this appendix, it has been shown how to calculate the higher order detection correlation effects induced by the search. This preliminary investigation indicates that, for the most part, the probabilities can be assumed to be essentially independent. However, for specific cases, such as asking for the time to the detection of the first target, this may prove insufficient. In such cases the approach given here can be used to provide more accurate probabilities. These multitarget correlations can also be important in the description of quitting rules. If, for example, the observer quits after finding the first target, the multiple-target correlations need to be computed to establish the appropriate rate.

APPENDIX D

NONSTATIONARY NEOCLASSICAL MODEL

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NONSTATIONARY NEOCLASSICAL MODEL

The neoclassical model describes detection as the average over possible search paths of a detection process that depends on the time on target. Only stationary processes have been described in detail, that is, search and detection processes whose parameters are independent of time. As noted in Section III, nonstationary models may have value. An extension of the neoclassical model to cover these cases is straightforward.

A search path can be described by a sequence of times spent on the target and off the target. In the neoclassical model, the time off the target is divided into two classes—wandering and attending to distracters—but this distinction is easily absorbed into the model. A particular search and detection process for a particular target can be described by a sequence of times (t_1, τ_1, t_2, τ_2, t_3, τ_3, ...) where t_1 is the time spent off target before the first visit to the target; τ_1 is the time spent on the target in the first visit, t_2 is the time spent off target prior to the second visit; τ_2 is the time spent on target during the second visit, etc. The probability of such a search path is:

$$dP(\{t_i, \tau_i\}) = \prod_{j=1}^{\infty} dt_j \, d\tau_j \, \tilde{W}_0^{(j)} \, e^{-\tilde{W}_0^{(j)} \tau_j} \left(\sum_{i_j} e_{i_j}^{(j)} \lambda_{i_j}^{(j)} e^{-\lambda_{i_j}^{(j)} t_j} \right) . \qquad \text{(D-1)}$$

In Eq. (D-1), the neoclassical model has been generalized to a nonstationary form for which the parameters can change for each visit. During the jth visit to the target, the time spent on target is exponentially distributed with a rate given by $\tilde{W}_0^{(j)} = W_0^{(j)} + J^{(j)} - J_0^{(j)}$. For example, one might model an observer by assuming that each subsequent visit to a target was shorter than the previous visit.

The terms inside the summation account for the time spent wandering and attending to distracters. The amplitudes have the same form as given in Eq. (IV-8) for the arrival time distribution[2] with superscripts indicating the values of the parameters on each visit:

$$\lambda_{1,3}^{(k)} = \frac{R^{(k)} + J_0^{(k)} \pm \sqrt{(R^{(k)} + J_0^{(k)})^2 - 4[S_0^{(k)} W^{(k)} + J_0^{(k)} S^{(k)}]}}{2} \quad \text{(D-2a)}$$

$$e_i^{(k)} = \frac{\lambda_j^{(k)} - J_0^{(k)} p^{(k)}(0) - S_0^{(k)} w^{(k)}(0) + p_0^{(k)}(0)(\lambda_i^{(k)} - R^{(k)})}{\lambda_j^{(k)} - \lambda_i^{(k)}} \ . \quad \text{(D-2b)}$$

The "initial conditions" for each period of off-target time have the same form in terms of the k-dependent parameters for all k > 1.

$$p_0^{(k)}(0) \ = 0; p^{(k)}(0) = \frac{J^{(k)} - J_0^{(k)}}{W_0^{(k)} + J^{(k)} - J_0^{(k)}}; \ w^{(k)}(0) = \frac{W_0^{(k)}}{W_0^{(k)} + J^{(k)} - J_0^{(k)}} \ . \quad \text{(D-3)}$$

Equations (D-1)–(D-3) describe the probability of an arbitrary path. To calculate the probability of detection on that path, one simply inserts the action of the detection process during each visit to the target. Consider, for example, an exponential detection process for which the detection rate varies from visit to visit, $\alpha_0^{(j)}$. Then the probability of detection is

$$P_D(t) = \int_{\text{Search path}} dP(\{t_k, \tau_k\})(1 - \exp(-\sum_k \alpha_0^{(j)} T_k(t))) \ , \quad \text{(D-4)}$$

where $T_k(t)$ is the time-on-target during the kth visit. For times before the beginning of the kth visit, $T_k(t) = 0$; after the end of the kth visit, $T_k = \tau_k$; during the kth visit, T_k increases linearly from 0 to τ_k. If t is located in the kth visit, the integrals over the future search path (m > k) parts can be done immediately; as noted in the derivation of the fictitious absorbing state, these integrals are identically unity. For any finite number of visits, only a finite number of integrals need to be performed.

The exponential factor in Eq. (D-4) is just the representation of the specific nonstationary exponential process used. For any other process, one just substitutes the appropriate form into the integrand.

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2 As mentioned in a previous note, if only a single point of interest is included the e_1 terms are identically zero and $\lambda_2 = s_0$; $e_3 = 1 - p_0(0)$.

$$P_D(t) = \int_{\text{Search path}} dP(\{t_k, \tau_k\})\, D(\{T_k(t)\}) \quad . \tag{D-4}$$

Equation (D-4) is a bit daunting. For a restricted visit model, however, it can be straightforwardly, if tediously, evaluated. The result for a single visit was given in Eq. (IV-10). For two visits, one has the result given in Eq. (D-6):

$$\begin{aligned}
P_D(t) \;=\; & P^{(1)}_{\text{visit}} + P^{(2)}_{\text{visit}} - P^{(1)}_{\text{visit}}\, P^{(2)}_{\text{visit}} \\
& - \sum_i e^{(1)}_i e^{-\lambda^{(1)}_i t} - \sum_i e^{(1)}_i \frac{\lambda^{(1)}_i}{\hat{W}_1 - \lambda^{(1)}_i} [e^{-\lambda^{(1)}_i t} - e^{-W_i t}] \\
& - \sum_i \sum_j e^{(2)}_j e^{(1)}_i \frac{\lambda^{(1)}_i W^{(1)}_0}{(\hat{W}_1 - \lambda^{(2)}_j)(\lambda^{(2)}_j - \lambda^{(1)}_i)} [e^{-\lambda^{(1)}_i t} - e^{-\lambda^{(2)}_j t}] \\
& + \sum_i \sum_j e^{(2)}_j e^{(1)}_i \frac{\lambda^{(1)}_i W^{(1)}_0}{(\hat{W}_1 - \lambda^{(1)}_i)(\hat{W}_1 - \lambda^{(2)}_j)} [e^{-\lambda^{(1)}_i t} - e^{-\hat{W}_1 t}] \\
& - \sum_i \sum_j e^{(2)}_j e^{(1)}_i \frac{\lambda^{(1)}_i \lambda^{(2)}_j W^{(1)}_0}{(\hat{W}_1 - \lambda^{(2)}_j)(\hat{W}_2 - \lambda^{(2)}_j)(\lambda^{(2)}_j - \lambda^{(1)}_i)} [e^{-\lambda^{(1)}_i t} - e^{-\lambda^{(2)}_j t}] \\
& + \sum_i \sum_j e^{(2)}_j e^{(1)}_i \frac{\lambda^{(1)}_i \lambda^{(2)}_j W^{(1)}_0}{(\hat{W}_1 - \lambda^{(2)}_j)(\hat{W}_2 - \lambda^{(2)}_j)(\hat{W}_1 - \lambda^{(1)}_i)} [e^{-\lambda^{(1)}_i t} - e^{-\hat{W}_1 t}] \\
& + \sum_i \sum_j e^{(2)}_j e^{(1)}_i \frac{\lambda^{(1)}_i \lambda^{(2)}_j W^{(1)}_0}{(\hat{W}_1 - \lambda^{(2)}_j)(\hat{W}_2 - \hat{W}_1)(\hat{W}_1 - \lambda^{(1)}_i)} [e^{-\lambda^{(1)}_i t} - e^{-\hat{W}_1 t}] \\
& - \sum_i \sum_j e^{(2)}_j e^{(1)}_i \frac{\lambda^{(1)}_i \lambda^{(2)}_j W^{(1)}_0}{(\hat{W}_1 - \lambda^{(2)}_j)(\hat{W}_2 - \hat{W}_1)(\hat{W}_2 - \lambda^{(1)}_i)} [e^{-\lambda^{(1)}_i t} - e^{-\hat{W}_2 t}]
\end{aligned} \tag{D-6}$$

$$- \sum_i \sum_j e_j^{(2)} e_i^{(1)} \frac{\lambda_i^{(1)} \lambda_j^{(2)} W_0^{(1)} W_0^{(2)}}{\hat{W}_1 (\hat{W}_1 - \hat{W}_2)(\hat{W}_1 - \lambda_i^{(1)}) (\hat{W}_1 - \lambda_j^{(2)})} e^{-\hat{W}_1 t}$$

(D-6 cont.)

$$- \sum_i \sum_j e_j^{(2)} e_i^{(1)} \frac{\lambda_i^{(1)} \lambda_j^{(2)} W_0^{(1)} W_0^{(2)}}{\hat{W}_2 (\hat{W}_2 - \hat{W}_1)(\hat{W}_2 - \lambda_i^{(1)}) (\hat{W}_2 - \lambda_j^{(2)})} e^{-\hat{W}_2 t}$$

$$- \sum_i \sum_j e_j^{(2)} e_i^{(1)} \frac{\lambda_i^{(1)} \lambda_j^{(2)} W_0^{(1)} W_0^{(2)}}{\lambda_i^{(1)} (\lambda_i^{(1)} - \hat{W}_1)(\lambda_i^{(1)} - \hat{W}_2)(\lambda_i^{(1)} - \lambda_j^{(2)})} e^{-\lambda_i^{(1)} t}$$

$$- \sum_i \sum_j e_j^{(2)} e_i^{(1)} \frac{\lambda_i^{(1)} \lambda_j^{(2)} W_0^{(1)} W_0^{(2)}}{\lambda_j^{(2)} (\lambda_j^{(2)} - \hat{W}_1)(\lambda_j^{(2)} - \hat{W}_2)(\lambda_j^{(2)} - \lambda_i^{(1)})} e^{-\lambda_j^{(2)} t} ,$$

where for notational compactness

$$\hat{W}_1 = W_0^{(1)} + J^{(1)} - J_0^{(1)} + \alpha_0^{(1)} ; \hat{W}_2 = W_0^{(2)} + J^{(2)} - J_0^{(2)} + \alpha_0^{(2)} \qquad \text{(D-7a)}$$

and the single visit probabilities are defined by

$$P_{visit}^{(1)} = \frac{\alpha_0^{(1)}}{W_0^{(1)} + J^{(1)} - J_0^{(1)} + \alpha_0^{(1)}} ; \; P_{visit}^{(2)} = \frac{\alpha_0^{(2)}}{W_0^{(2)} + J^{(2)} - J_0^{(2)} + \alpha_0^{(2)}} . \qquad \text{(D-7b)}$$

The P_∞ for this example is as expected given by

$$P_\infty = P_{visit}^{(1)} + P_{visit}^{(2)} - P_{visit}^{(1)} P_{visit}^{(2)} . \qquad \text{(D-8)}$$

In the stationary case, the exponents become degenerate (that is, $W_0^{(1)} = W_0^{(2)}$, etc.). The proper result is obtained by taking the appropriate limit. The result includes factors of the time, t, multiplying some of the exponentials.

Although Eq. (D-6) is cumbersome and the K = 3 case would be appalling, any fixed K case could be worked out or a nonstationary version of the Monte Carlo simulation procedure described above could be used. The point is not to propose Eq. (D-6) as the definitive search and detection model, but simply to illustrate the flexibility, robustness, and power of the neoclassical framework.

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Jeffrey Nicoll

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Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311-1772

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13. ABSTRACT *(Maximum 180 words)*

Search is currently modeled for DoD applications by a singe exponential function. The two adjustable parameters are the time constant, t, characterizing the exponential; and the long time detection probability, P_∞. Deficiencies of the classical model are: (1) human performance data cannot typically be fit with a single exponent model; the probability of detection for short times is less than that predicted by the classical model; (2) the effects of multiple targets and clutter can only be included by adjusting the two-model parameters, which is performed in an ad hoc manner and overconstrains the model. This paper introduces a neoclassical model that includes three processes: (1) attending to the target; (2) random wandering around the scene; and (3) attending to other targets/clutter. An expression involving three exponents associated with the three processes is derived and special cases are described. The new model provides uniform treatment of multiple targets and false detections and allows for the separate descriptions of multiple times scales within the search process. Searches can be separated into single region, field-of-view search, and multiple region, field-of-regard search. Field-of-view search can be further subdivided into long searches during which the observer may examine many targets and short searches which are completed after a few target examinations.

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